



Modelling Biogas Production from Organic Waste Substrates Using the Gompertz Equation: Parameter Estimation and Methane Composition Analysis

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Abstract

Biogas technology converts organic trash (substrate) into energy while also enhancing the environment and human well-being. Design of bioprocess for biogas production requires empirical/kinetic models for proper sizing of equipment and control of process variables. This work focuses on the determination of Gompertz model parameters for sheep excrement, leftover tomatoes, chicken droppings, and leftover fluted pumpkin leaves as substrates. The biogas produced had an average composition of 65.34 mol% methane and 22.81 mol% carbon dioxide; other components included hydrogen 7.49 mol% and nitrogen 3.39 mol%, as well as tiny amounts of water vapor and carbon monoxide. Hydrogen sulphide was not found. The cumulative volume–time data obtained was fitted into the Gompertz model founded on non-linear regression analysis with Microsoft Excel's Solver program. Gompertz model provided high cross-correlation coefficients of 0.997, 0.996, 0.997 and 0.993 for tomato waste, chicken droppings, Fluted Pumpkin Leaves Waste, and sheep manure respectively.

Keywords: Empirical model, Gompertz model, Non-linear regression, substrate, biogas production.

1. INTRODUCTION

Methane is the main component of biogas, which is a gas produced by the anaerobic breakdown of organic wastes like vegetables, plants, crop residues, and human and animal wastes. It also contains contaminants like CO₂, N₂, H₂ and H₂S [1]. Recent years have seen a significant increase in interest in biogas reactors due to the need to provide a renewable energy source to lessen reliance on fossil fuels, which are driving global warming. According to one of the reports [2], anaerobic digestion of a feedstock into biogas is a multi-covered process affected by many variables such as substrate, temperature, pH, and microbial action. Biogas prediction and modeling is an important part of this process since it ensures biogas optimization, energy protection, and environmental protection. Mathematical models have for decades been the sole means of estimating biogas production and have been used to understand the mechanisms behind anaerobic digestion. Several types of biogas reactors pigeon-holed into batch, sequencing batch and continuous have emerged to support the

tendency towards system approaches and greater investigation into the biochemistry and functional characteristics of biogas reactors. Thorough explanation of the phenomena takes in all applicable mathematical models to the design of a reliable biogas reactor and its performance appraisal. The literature includes a variety of mathematical models such as stoichiometric models for biogas generation, predictive models for cumulative biogas metaphors as well as reaction kinetics that consider substrate threshold, product inhibition, and other determinants. The empirical models such as Gompertz and logistic models that enable the prediction of biogas yields from different substrates proved especially useful according to recent reviews [3][4]. They are based on statistical relationships between the input parameters and the outputs with biogas yields and also provide a quick and easy way of estimating the outputs. The kinetic models have also been applied for modeling of biogas production based on agricultural and municipal wastes, particularly the Monod model and Contois model [5][6].

These models are kinetically based and describe the major biochemical processes involved in anaerobic digestion together with the growth of microbes [7][8][9]. Machine Learning and Deep Learning models have become practical tools for estimating biogas generation [10][11]. Computational Intelligence models have the ability to recognize trends in past data and extrapolate from novel data and this makes them an optimal solution for predicting biogas yields. There have been computer models predicting biogas output from various sources and these have been shown to work [12]. Other studies have explored the use of hybrid models, combining empirical, kinetic, and mechanistic approaches to predict biogas production [13]. Computational Intelligence models have successfully reached the aim of predicting the amount of biogas produced, yet there are still difficulties encountered. These include model validation requirements such as data quality, and model complexity issues where precision and decision support derive from an interpretable but simple model [13].

The relationship of computational intelligence models integrated with sensor systems and online monitoring systems has also been investigated [14]. These combined systems allow monitoring and prediction of biogas production so that effective control and management processes can be carried out [15]. Machine learning models such as k nearest neighbors regression, logistic regression, support vector machine, random forests and extreme gradient boosting. [16] generated two robust algorithms aimed at modelling bio digestion systems as a function of the most influencing parameters encompassing least square support vector machines (LSSVM) and fuzzy inference systems based on adaptive networks (ANFIS). Multiple statistical analyses were used to evaluate the models for both the actual values and the model results. [17] carried out the creation of the Adaptive Neuro Fuzzy Inference System (ANFIS) and Artificial Neural Network (ANN) to forecast the amount of biogas. The process factors included the type of bioreactor, pH, FOS/TAC ratio, and temperature (°C). In [18], a robust anaerobic co-digestion (AcoD) model for predicting biogas was created using deep learning (DL). They proposed a hybrid deep learning architecture, DA-LSTM-VSN, in which a dual-stage attention (DA)-based long short-term memory (LSTM) network was combined with variable selection networks (VSNs). To improve model predictability, they also performed a hyperparameter optimization. [19] employed an MLP (Multi-layer Perceptron) neural network to create a required biogas property predictor model. [20] built a three-layer artificial neural network (ANN) using nonlinear regression models to estimate biogas production from an anaerobic hybrid reactor. [21] proposed a novel model based on spiking neural network cubes to model the chemical processes that go on in a digester to produce usable biogas. [22] employed three modeling approaches - Fuzzy Mamdani Model (FMM), Artificial Neural Network (ANN), and

Response Surface Methodology (RSM) - to optimize biogas production from various combinations of poultry waste and cow dung in a modular bioreactor system. [23] designed a biogas production management system to regulate biogas output by adjusting feedstock inputs to the anaerobic digestion process in response to fluctuations in renewable energy supply. The system comprises three key components: a predictive model for anaerobic digestion, a parameter estimation module, and a feedstock control mechanism. The feasibility of ML models was verified by [24] for routine monitoring of data from industrial-scale biogas plants treating food waste (FW) to predict biogas yield. They argued that monitoring indicators and their frequency should be reviewed to create more sophisticated machine learning (ML) models and increase system productivity. In [25], a hybrid machine learning method that includes random forest (RF) and long short-term memory (LSTM) analysis was employed. The method was used to find out the factors that impact biogas production from a biogas plant and try to optimize the prediction of biogas outputs. [26] integrated artificial neural network (ANN) models with modified Gompertz models (MG) to predict the cumulative biogas and methane yields from anaerobic digestion of some organic wastes. On the other hand, [27] sought to quantify biogas production from faecal sludge using a three-layer backpropagation neural network [BPNN], granules and tank bottom sludge that resulted from the use of a continuous stirred tank. [28] conducted a comprehensive assessment of different sources of biomass for biogas production and investigated the influence of co-digestion on biogas production and the technology involved.

2. MATERIALS AND METHOD

This section describes the equipment, resources and the materials employed as well as the methodological tools and techniques. A thorough account of the experimental apparatus, mode of data collection and models employed for optimizing reliability and the reproducibility of the results achieved are provided.

2.1 Materials

The Equipment employed are the water displacement type digester (digester bottle, gas collector bottle and a calibrated bottle), weighing balance, vacuum pump, rubber tubes, plastic funnel, bucket, beaker, glass rod and measuring cylinder, sludge used for seeding was extracted from a landfill close to the Department of Petroleum and Natural Gas

Processing, Petroleum Training Institute Effurun, Delta State Nigeria. Wasted tomatoes, leftover pumpkin leaves, chicken manure, and sheep dung are the

substrates used, as they are obtained within the Effurun metropolis, Delta State, Nigeria.

2.2 Experimental Set-up

The water displacement method was utilized to

measure biogas production. The experimental setup consisted of a biodigester bottle, a gas collector bottle filled with water, and a graduated container. To prepare the feedstock, 100g of substrate was mixed with 100ml of water (1:1 ratio) and 20ml of inoculum. The mixture is then loaded into the biodigester bottle and connected to the gas collector and graduated container as shown in Fig. 1.

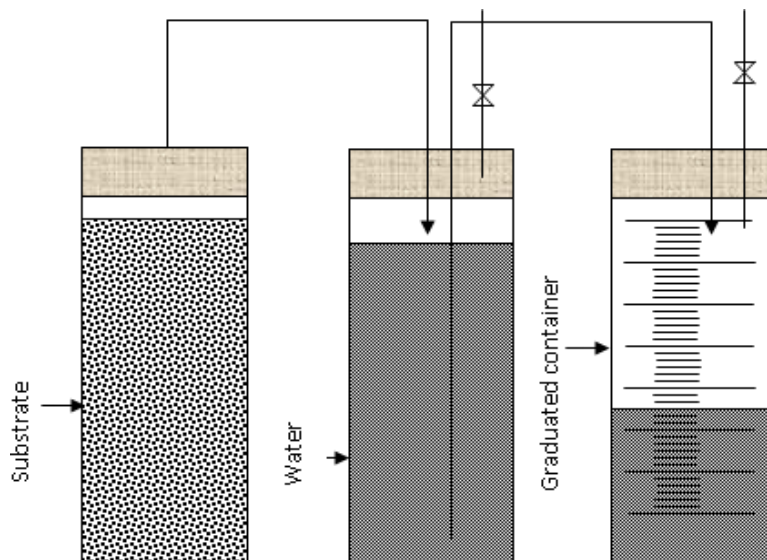


Fig. 1: Setup for Water Displacement Method

The amount of water that was forced into the graduated container from the gas collector was used to calculate the amount of biogas that was generated. The generation of biogas was measured every day for 22 days at 24-hour intervals. The gas generated was examined using the Gas Chromatograph ASXL-FID at the quality control laboratory (LPG and Gases) of the Warri Refining and Petrochemical Company Ltd (WRPC).

2.3 Models Employed

The kinetics of microbial growth have been studied using a variety of models. The key elements that affect the pace of bacterial growth are the focus of all models. The Malthusian model, sometimes referred to as Malthus law or exponential law [29], is the most basic model for microbial cell multiplication. It is represented mathematically in Eqn. (1).

$$\frac{dC_x}{dt} = \mu C_x \quad (1)$$

Where, μ is the Specific microbial growth rate, d^{-1} , C_x is

the Microbial concentration, mg/L.

The specific microbial growth rate μ is not constant but dependent on many parameters such as microbial concentration, substrate concentration, pH, concentration of inhibitors etc. Eqn. (2) shows the specific microbial growth rate.

$$\mu = \frac{\mu_m C_s}{K_s + C_s} \quad (2)$$

The deterministic raw kinetic parameters model [30], the growth rate of microorganisms and the half velocity constant – predict when biological activity will peak and when it will stop. Eqn. (3) is used to calculate the rate of substrate utilization (r_s).

$$r_s = \frac{\mu_m C_x C_s}{Y(K_s + C_s)} \quad (3)$$

The Gompertz model [31] represented by Eqn. (4), is the most popular model that biogas production researchers have tried to fit empirical data with.

$$B(t) = B_{max} \cdot \exp \left[- \exp \left(\frac{R_m e}{B_{max}} (\lambda - t) + 1 \right) \right] \quad (4)$$

$B(t)$ is the Cumulative biogas (or methane) production at time t (mL, L, or m³), B_{max} is the maximum biogas production potential (mL, L, or m³), R_m is the maximum biogas production rate (mL/day, L/day, or m³/day), λ is the lagged phase time (days), t is the cumulative time for biogas production (hr), e is the Euler's number (approx. 2.718).

The Modified Logistic model in Eqn. (5) is expressed,

$$V = \frac{A}{1 + \exp \left(\frac{4R_{max}(\lambda - t)}{A} + 2 \right)}$$

Where μ_m is the maximum specific microbial growth rate, d⁻¹, K_S is the half-saturation constant (Monod's constant), mg/L, C_s is the substrate concentration, mg/L,

4. RESULTS AND DISCUSSION

The daily cumulative volume of biogas produced from each of the four substrates was recorded and summarized in Table 1. The data is visualized in Fig. (2). The composition of the biogas is analyzed, and the results are presented in Table 2.

Table 1: Water Displacement Method Biogas Production Results Days

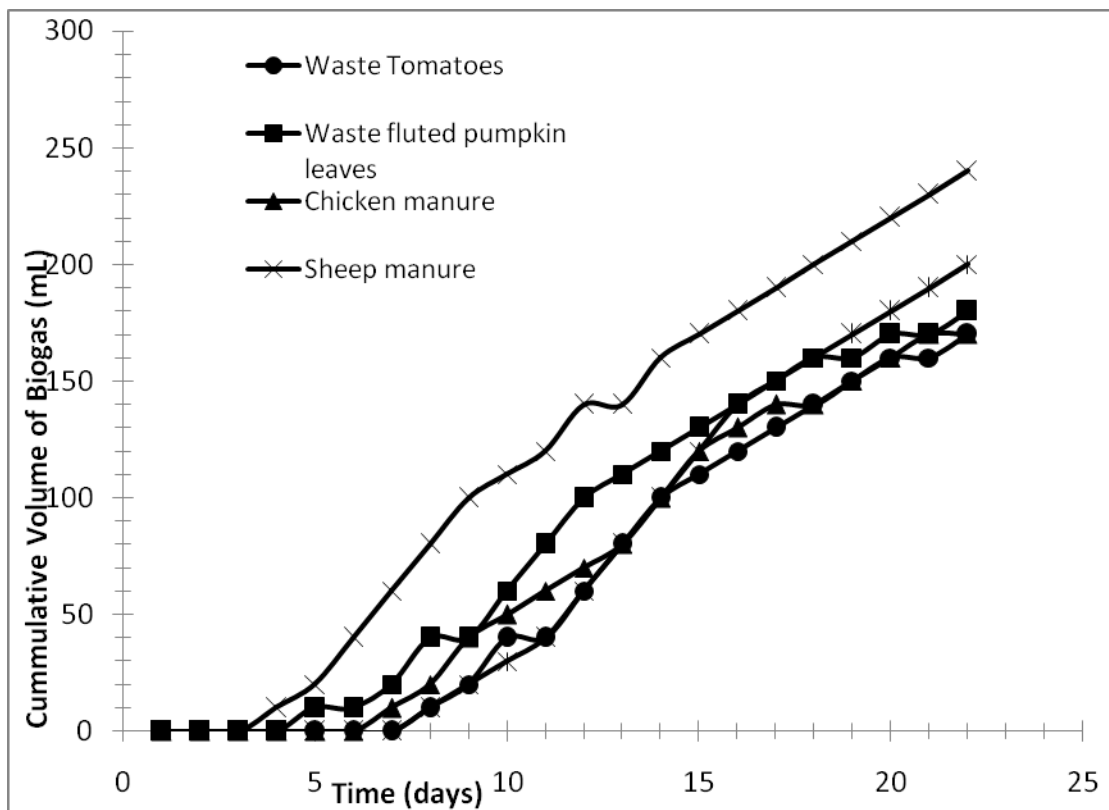
S/N	Tomatoes Waste (mL)	Fluted Pumpkin Leaves Waste (mL)	Chicken Droppings (mL)	Sheep Manure (mL)
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0.1
5	0	0.1	0	0.2
6	0	0.1	0	0.4
7	0	0.2	0.1	0.6
8	0.1	0.4	0.2	0.8
9	0.2	0.4	0.4	1.0
10	0.4	0.6	0.5	1.1
11	0.4	0.8	0.6	1.2
12	0.6	1.0	0.7	1.4
13	0.8	1.1	0.8	1.4
14	1.0	1.2	1.0	1.6
15	1.1	1.3	1.2	1.7
16	1.2	1.4	1.3	1.8
17	1.3	1.5	1.4	1.9
18	1.4	1.6	1.4	2.0
19	1.5	1.6	1.5	2.1
20	1.6	1.7	1.6	2.2
21	1.6	1.7	1.7	2.3

Table 2: Results for biogas analysis using gas chromatography

S/N	Component	Mole %
1	Methane	65.34
2	Carbon (IV) Oxide	22.81
3	Water	Trace
4	Nitrogen	3.39
5	Hydrogen	7.49
6	Carbon (II) Oxide	Trace

The Cumulative volume for the Waste Tomatoes substrates, Waste Fluted Pumpkin Leaves substrates,

Chicken Droppings substrates and Sheep Manure substrates is shown in Fig. 2.

**Fig. 2:** Cumulative volume for the substrates

The Data fitting into the Gompertz Model for the Waste Tomatoes substrates, Waste Fluted Pumpkin Leaves substrates, Chicken Droppings substrates and Sheep Manure substrates is described in Fig. 3.

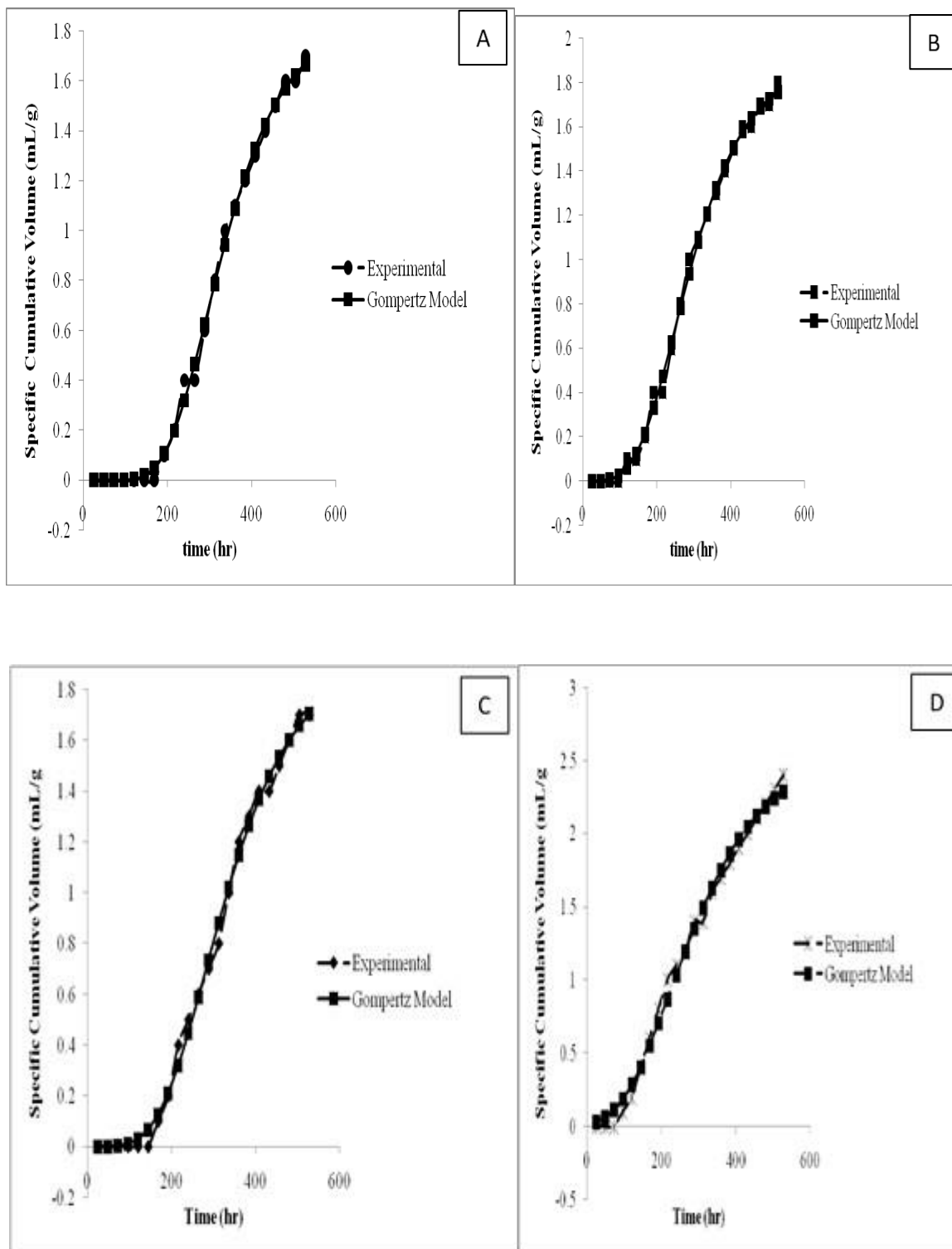


Fig. 3: Data fitting into Gompertz Model (a) Waste Tomatoes (b) Pumpkin Waste Fluted Pumpkin Leaves (c) Chicken Droppings (d) Sheep Manure

The goodness of fit for the substrates tested in Table 3

Table 3: Model parameters and goodness of fit for the substrates

S/N	Substrate	A (mL/g)	Rmax (mL/g.hr)	Λ (hr)	R ²	MSE
1	Waste Tomatoes	1.838	0.0068	8.1	0.997	0.00048
2	Waste Fluted Pumpkin Leaves	1.891	0.0066	6.0	0.997	0.00052
3	Chicken droppings	1.922	0.0061	6.9	0.996	0.00077
4	Sheep manure	2.555	0.0067	3.75	0.993	0.00238

The combustion test carried out shows that the gas produced is biogas. It burnt with a blue flame without soot. It was able to raise the temperature of 200ml of water from 27 °C to 60 °C in 10 minutes. In comparison with LPG for the same volume of water and time duration (10 minutes), it raised the temperature from 27 °C to 80 °C. This implies that LPG has a higher calorific value than biogas.

CONCLUSION

In the Waste Tomatoes substrates, Waste Fluted Pumpkin Leaves substrates, Chicken Droppings substrates and Sheep Manure substrates, the data fitted very well into Gompertz model with very high values of R². Time-course profile of cumulative biogas production for all substrates shows similarity with the existing curves found in literature. The Gompertz model is identified as a suitable empirical model based on the obtained results for predicting rate of biogas production for the substrates.

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