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Original Research Paper

Using a Direct Multiple Shooting Method to Control a Quadrotor

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Abstract

Optimal control techniques have become increasingly important for the autonomous operation of unmanned aerial vehicles, particularly quadrotors, due to their nonlinear dynamics and underactuated structure. In this paper, a direct multiple shooting method is investigated for solving optimal control problems and applied to the energy-efficient control of a quadrotor. The multiple shooting method divides the optimisation horizon into several smaller intervals and transforms the original optimal control problem into a nonlinear programming (NLP) problem by introducing additional state variables and continuity constraints. The resulting NLP is solved by computing the zeros of the Lagrangian Jacobian using Newton's method. First, the theoretical foundations of the direct multiple shooting approach are presented. Then, a numerical example is used to demonstrate the effectiveness of the method. Finally, the proposed approach is applied to a quadrotor control problem focused on minimising energy consumption while ensuring accurate trajectory tracking. Simulation results indicate that the direct multiple shooting method provides stable convergence, improved numerical robustness, and efficient control performance for quadrotor systems.

Keywords: optimal control, multiple shooting method, nonlinear programming, Newton's method, quadrotor, energy minimisation.

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1. INTRODUCTION

Unmanned aerial vehicles (UAVs) have received considerable attention recently due to their wide range of applications, including surveillance, inspection, mapping, environmental monitoring, and delivery services. Among the various UAV configurations, quadrotors are particularly attractive because of their simple mechanical structure, vertical take-off and landing capabilities, and high manoeuvrability.

Despite these advantages, quadrotors exhibit highly nonlinear and coupled dynamics. Their control therefore requires advanced methods capable of handling nonlinearities, constraints, and performance objectives simultaneously. Optimal control provides a systematic framework for generating control inputs that minimise a given performance criterion while satisfying system dynamics and operational constraints.

Traditional indirect methods for optimal control rely on the derivation of necessary optimality conditions, resulting in two-point boundary value problems that can be difficult to solve numerically. Direct methods overcome these difficulties by discretising the control problem and converting it into a finite-dimensional optimisation problem.

Among direct methods, the direct multiple shooting approach has emerged as an effective technique due to its superior numerical stability compared with single shooting. By partitioning the time horizon into smaller subintervals and introducing intermediate state variables, multiple shooting improves convergence properties and reduces sensitivity to initial guesses.

The objective of this paper is to present the direct multiple shooting method and demonstrate its application to the optimal control of a quadrotor. The proposed framework minimises control energy while steering the vehicle toward its desired states.

The contributions of this work are:

1. Presentation of the direct multiple shooting formulation for optimal control problems.
2. Transformation of the optimal control problem into a nonlinear programming problem.

3. Solution of the NLP using Newton’s method.
4. Application to energy-efficient quadrotor control.
5. Numerical validation through simulation examples.

2. Optimal Control Problem Formulation

Consider the nonlinear dynamic system.

$$\dot{x}(t)=f(x(t),u(t),t),$$

where

- $x(t)\in \mathbb{R}^{n}$ denotes the state vector,
- $u(t)\in \mathbb{R}^{m}$ denotes the control vector,
- $f(\cdot)$ is a nonlinear differentiable function.

The optimal control problem consists of minimising the performance index.

$$J=\Phi(x(t_f))+\int_{t_0}^{t_f}L(x(t),u(t),t),dt,$$

subject to

$$\dot{x}(t)=f(x(t),u(t),t),$$

$$x(t_0)=x_0,$$

and possible state and control constraints

$$g(x(t),u(t))\leq 0.$$

The terminal state may be fixed or partially constrained depending on the application.

3. Direct Multiple Shooting Method

3.1 Principle

The interval

$$[t_0,t_f]$$

is divided into N subintervals:

$$t_0<t_1<t_2<\cdots<t_N=t_f.$$

Instead of integrating the system over the entire interval from a single initial condition, multiple shooting introduces independent initial states.

$$s_k=x(t_k), \quad k=0,\ldots,N-1.$$

For each interval $[t_k,t_{k+1}]$, an initial value problem is solved:

$$\dot{x}_k(t)=f(x_k(t),u_k(t)),$$

$$x_k(t_k)=s_k.$$

The solution over each interval is represented by

$$x_k(t)=\phi(t,t_k,s_k,u_k),$$

where ϕ denotes the numerical integration operator.

To ensure continuity across intervals, matching conditions are imposed:

$$s_{k+1}-\phi(t_{k+1},t_k,s_k,u_k)=0.$$

These continuity equations constitute additional equality constraints.

3.2 Discretisation of Controls

Controls are parameterised using piecewise constant values:

$$u(t)=u_k,\quad t\in[t_k,t_{k+1}].$$

The optimisation variables become

$$z=[s_0,s_1,\ldots,s_N,u_0,u_1,\ldots,u_{N-1}]^T.$$

The optimal control problem is thus transformed into a finite-dimensional optimisation problem.

4. Nonlinear Programming Formulation

After discretisation, the problem can be written as

$$\min_{z} J(z)$$

subject to

$$c(z)=0,$$

where $c(z)$ contains all continuity and boundary constraints.

The associated Lagrangian is

$$J(z)+\lambda^Tc(z),$$

where λ is the vector of Lagrange multipliers.

The first-order optimality conditions are

$$\nabla_z \mathcal{L}(z,\lambda)=0,$$

$$c(z)=0.$$

These conditions form a nonlinear system of equations.

5. Newton’s Method for the NLP

To solve the Karush-Kuhn-Tucker (KKT) conditions, Newton’s method is employed.

Define

$$F(y)=\begin{bmatrix}\nabla_z \mathcal{L}(z,\lambda)\\ c(z) \end{bmatrix},$$

where

$$y= \begin{bmatrix} z\\\lambda \end{bmatrix}.$$

At iteration k, Newton’s method computes

$$JF(y_k)\Delta y_k=-F(y_k),$$

where $JF(y_k)$ is the Jacobian matrix of F.

The update is

$$y_{k+1}=y_k+\Delta y_k.$$

Iterations continue until

$$|F(y_k)|<\varepsilon,$$

for a prescribed tolerance ε.

The multiple shooting formulation typically leads to sparse Jacobian matrices, which can be exploited to improve computational efficiency.

6. Numerical Example

To illustrate the direct multiple-shooting method, consider the system.

$$\dot{x}(t)=-x(t)+u(t),$$

with initial condition

$$x(0)=1,$$
and terminal condition
$$x(1)=0.$$
The objective is to minimise
$$J=\int_0^1 u^2(t) dt.$$

The interval ([0, 1]) is divided into ten equal subintervals.
The controls are assumed piecewise constant on each interval.
The resulting NLP contains:

- State variables at interval boundaries,
- Control parameters,
- Continuity constraints.

Newton's method is applied to solve the resulting KKT system.
Simulation results demonstrate smooth state convergence toward the target state while minimising control effort. The continuity constraints are satisfied within numerical tolerance, confirming the effectiveness of the multiple shooting approach.

7. Quadrotor Dynamic Model

7.1 Coordinate Frames

The quadrotor motion is described in:

1. An inertial frame (I),

2. A body-fixed frame (B).

The position vector is
$$p=[x,y,z]^T.$$
The attitude is represented by Euler angles.
$$\eta=[\phi,\theta,\psi]^T,$$
where:

- (ϕ): roll angle,
- (θ): pitch angle,
- (ψ): yaw angle.

7.2 Translational Dynamics

The translational equations are
R(\phi,\theta,\psi)
\begin{bmatrix} 0\0\T\end{bmatrix}

+
\begin{bmatrix}
0\
0\
-mg
\end{bmatrix},
]
where

- (m) is the mass,
- (g) is gravitational acceleration,
- (T) is the total thrust,
- (R) is the rotation matrix.

7.3 Rotational Dynamics

The rotational dynamics are
\tau,]
where

- I is the inertia matrix,
- ($\omega=[p,q,r]^T$) is the angular velocity vector,
- ($\tau=[\tau_\phi,\tau_\theta,\tau_\psi]^T$) is the torque vector.

7.4 Control Inputs

The quadrotor possesses four control inputs:
$$u=[T,\tau_\phi,\tau_\theta,\tau_\psi]^T.$$
These inputs are generated by varying the angular velocities of the four rotors.

8. Energy-Optimal Quadrotor Control

The objective is to move the quadrotor from an initial state
$$x(0)=x_0$$
to a desired final state
$$x(t_f)=x_f$$
while minimising energy consumption.
The performance index is
$$J=\int_0^{t_f} u^T(t)Ru(t)dt,$$
where (R) is a positive definite weighting matrix.
The optimisation problem is therefore
$$\min_{u(t)} J$$
subject to:
$$\dot{x}=f(x,u),$$
$$x(0)=x_0,$$
$$x(t_f)=x_f.$$
The direct multiple shooting method converts this problem into a nonlinear programming problem.

9. Simulation Results

The proposed approach was implemented using a direct multiple shooting framework combined with Newton-based optimisation.
The simulation setup considered:

- Initial position: (0, 0, 0) m
- Final position: (5, 5, 5) m,
- Final time: 10 s,
- Twenty shooting intervals.

The obtained trajectory was smooth and satisfied all boundary conditions.
The control inputs remained within admissible limits while achieving the desired manoeuvre.
Compared with single shooting methods, the direct multiple shooting approach exhibited the following:

- Improved convergence,
- Reduced sensitivity to initial guesses,
- Better numerical stability,
- Accurate satisfaction of continuity constraints.

Furthermore, the energy objective produced moderate control actions, resulting in reduced actuator effort and smoother flight trajectories.

These results demonstrate the suitability of the method for practical quadrotor optimal control applications.

10. Discussion

The direct multiple shooting method offers several advantages for nonlinear optimal control problems.

First, by dividing the time horizon into smaller intervals, integration errors do not accumulate significantly over long horizons. Second, the introduction of intermediate states improves convergence properties and allows the optimiser to correct local trajectory errors.

For quadrotor systems, which are characterised by nonlinear and coupled dynamics, these properties are particularly beneficial. The method effectively handles trajectory optimisation while maintaining numerical robustness.

The principal disadvantage is the increase in the number of optimisation variables due to the additional shooting states. However, modern sparse NLP solvers can efficiently exploit the resulting problem structure.

11. Conclusion

This paper presented a direct multiple shooting method for solving optimal control problems and demonstrated its application to energy-efficient quadrotor control.

The method partitions the time horizon into multiple intervals, solves an initial value problem on each interval, and enforces continuity through matching constraints. This formulation transforms the original optimal control problem into a nonlinear programming problem.

The resulting KKT system was solved using Newton's method. A numerical example illustrated the implementation procedure, and the method was subsequently applied to a quadrotor trajectory optimisation problem.

Simulation results showed that the direct multiple shooting approach provides accurate trajectory generation, reduced energy consumption, and strong numerical stability. These characteristics make it a promising framework for advanced UAV control and trajectory optimisation.

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