

Mathematical Modeling of Goldschmidt and Bowen's Crystallization of Mafic Magma: Implication for the Crystallization of Basaltic Rocks

¹Achuenu Ifeanyi, ²Iekmang.C. Isahand, ²Hyeladi Dibal ³Geoffrey Micah Kumleng

¹Department of Mining Engineering, University of Jos

²Department of Geology, University of Jos

³Department of Mathematics, University of Jos

Email. achuenuifeanyi@gmail.com

Abstract

This study presents a comprehensive mathematical framework to model the crystallization processes of mafic magma, integrating Goldschmidt and Bowen's theories within a thermodynamic context. By employing advanced mathematical tools such as metrical matrices, set notation, and complex analysis, the research aims to elucidate the distribution and substitution of chemical elements during mineral formation from magma. The model addresses the gap between Bowen's reaction series and Goldschmidt's elemental substitution rules, establishing a coordinate geometric relationship between continuous and discontinuous reaction series. The approach facilitates the calculation of mineral compositions and their evolution throughout the crystallization process, with applications to basaltic rocks and associated mineralogy. The methodology incorporates thermodynamic principles, ionic radii, electronegativity, and complex lattice interactions, enabling a holistic understanding of mineral formation and elemental distribution. The findings offer valuable insights for mineral exploration, petrology, and geochemical modeling, advancing the quantitative understanding of igneous rock genesis.

Keywords: Mathematical modeling, Mafic magma, Crystallization processes, Elemental substitution Goldschmidt

Accepted 15/5/2025

Published 30/5/2025

1.0: INTRODUCTION

Using mathematics in fields like physics, chemistry, and engineering has led to many successful inventions that improve human life. However, in geology, the use of mathematics hasn't been as effective. This research focusses on the challenges of using math to understand how chemical elements are substituted and distributed in igneous rocks during the entire crystallisation process, from start to finish, while considering changes in thermodynamics. The differences between Bowen's and Goldschmidt's ideas about how these elements behave during crystallisation will be explored through mathematical principles related to thermodynamics. In simpler terms, various minerals come together in specific arrangements to form rocks, and these minerals are made up of different chemical elements that are bonded together and have a unique chemical makeup.

The gap between Bowen's and Goldschmidt's concepts concerning the problem of elemental

substitution and distribution of chemical elements in rocks throughout the time of crystallisation, from the beginning to the end of crystallisation would be investigated using mathematical context with respect to thermodynamic principles.

Geometrically, several minerals are arranged in matrices and aggregate together to form rocks, while chemical elements with various properties bond to create minerals, which are defined as inorganic substances with distinct chemical compositions. Goldschmidt classified these elements according to their class of materials, such as siderophile, lithophile and chalcophile elements, as well as atmophile elements. But Mendeleev (1914) arranged these elements in a periodic table according to size and atomic mass. According to modern periodic law, atomic number increases from left to right and from top to bottom of the periodic table. Therefore the size and electronegativity of elements increase across the period,

and the size increases with decreasing electronegativity down the group. This pattern of modern periodic law would be used to explain how ions would enter into the lattice of a growing crystal during crystallisation of silicate magma as a function of size and electronegativity. During crystallisation of magma, temperature falls with increasing content of silicon in the magma. This forms two series of reactions (Bowen's), one by interaction with magma, with the first mineral formed, to form minerals of different chemical composition, but with the same crystallographic structure. This is the case of a solid solution in the magma. While the other, the first crystal to form, initially interacts with the magma to form a solid with a different crystallographic structure and distinct chemical composition. This is the case of fractional crystallisation in the magma.

The terms solid solution and fractional crystallisation are used simultaneously in this research to define the two series of reactions as stipulated by Bowen's reaction series.

Mathematically, the formation of rock-forming minerals from magma \mathfrak{R} depends on the numerical values of each of the minerals under thermodynamic change. Any stable materials depend on the numerical values for those materials to exist at that certain ambient temperature, especially at a room temperature of 25°C (298 K) at 1 atm pressure. The problem involving the distribution of chemical elements and elemental substitutions in rock during crystallisation of magma \mathfrak{R} , from the beginning of crystallisation to the end of crystallisation, would be postulated to some extent using combined Goldschmidt and Bowen's concept with respect to electronegativity and radius ratio, which states that, for ions of various electronegativities and sizes to compete for the space in a lattice of a growing crystal;

1. The one with more electronegative would preferentially enter into the lattice of the growing crystal of silicate radical because the nucleus of silicate radical is electronegative provided modern periodic law is obeyed and
2. That ions of closest radius (δ) to the nucleus of formed crystal substitute themselves before the ion of other radius ratio according to Wood (2003).

1.1: Statement of Problems

Bowen (1928) utilized temperature and time to establish relationships among essential rock-forming minerals, while Goldschmidt (1937) used ionic charge and size to explain ionic substitutions in crystals (Diadochi).

1. In a mathematical context, a significant challenge arises as there was no documented mathematical framework complementing the explanation of Bowen's reaction series, which encompasses the continuous (solid

solution) and discontinuous (fractional crystallization) reaction series.

2. Both Bowen and Goldschmidt lacked a mathematical connection in their rules, hindering a complete mathematical explanation of rock-forming minerals and their crystallization processes.

3. Specifically, the absence of a coordinate geometric relationship between the two reaction series (continuous and discontinuous) presented a notable obstacle to their comprehensive understanding.

4. To achieve a holistic understanding, a need exists to bridge the gap between Bowen's and Goldschmidt's rules with a mathematical foundation to comprehensively explain the formation of rock-forming minerals and their reactions.

1.2: Aim and Objectives

This research focuses on bridging the gap between Bowen's and Goldschmidt concepts concerning *the "problem of elemental substitution and distribution of chemical elements in rocks throughout the time of crystallization from the beginning to the end of crystallization with mathematical foundation under thermodynamic change"* and two objectives would be established as followed;

1. With a mathematical context, Bowen's and the Goldschmidt combined concept would be established using proper computations of numerical values of minerals under thermodynamic conditions which bridge the gap between the two combined rules
2. Mathematical methods such as Metrical Matrix and Set notion would be used to compute and define all the minerals and rocks in a complex plane of magma \mathfrak{B} and space.

1.3: Justifications of Research

1. The combined concepts of Bowen's and Goldschmidt would provide a concise explanation of rock forming minerals, aiding students in mineralogy and petrology to comprehend rock formation during magma crystallization. It utilizes mathematical methods and thermodynamic principles for the computation of numerical values of minerals into the complex plane and space. .

2. This research when completed would be a concise explanation of rock forming minerals from the complex plain of magma \mathfrak{R} and would be one of the simplified concept to study mineralogy and petrology as well as its application to mineral exploration.

3. Bowen's (1928) uses Y-shape to explain the progressive crystallization of minerals from the melt as temperature drops. This describes a diagram showing two types of reaction series: a continuous plagioclase

series on the left, a discontinuous ferromagnesian series on the right.

4. This Concept is a mathematical foundation that would be used to bridge the gap between Bowen's and Goldschmidt rules concerning the problem of elemental substitution and distribution of chemical elements in rock throughout the time of crystallization from the beginning to the end of crystallization

1.4: Assumptions

1. Rocks' chemical, mineral compositions and crystallization temperatures can be measured.
2. Thin section examination, geochemical analysis, melting and freezing of existing rocks, or synthesis from chemical reagents.
3. Assumptions for crystal formation during magma cooling
4. Magma homogeneity under constant thermodynamic conditions.
5. Magma fractionates into different domains (fractionation) under changing thermodynamic conditions.
6. \mathfrak{M} is recharged magma (magma domain), \mathfrak{R} is resident magma (homogeneous magma), and Z_0 is the nucleation point in magma domain \mathfrak{M} .
7. Crystal Z forms at certain thermodynamic and electrolytic conditions from $f(\mathfrak{M}) = Z_0$, and magma in a homogeneous form \mathfrak{R} behaves as a weak electrolyte.
8. Atomic nucleation Z_0 in the recharged magma domain requires appropriate atoms or ions to chemically bond and form a stable crystal nucleus Z under constant thermodynamic conditions.

9. The algebraic sum of anionic radical and cation charges equals zero during crystallization, balancing charges as temperature drops.

1.5: Scope of The Study

Research work encompasses major minerals: olivine, pyroxene, amphibole, mica, and feldspar, and associated rocks: granite, basalt, andesite, and trachyte.

1.5: Regional Geologic Setting

The study area is composed of rocks of the Precambrian to mid Cambrian and Jurassic northern Nigeria crystalline shield (Schoeneich, 1991). The basement complex within this shield is of Precambrian to mid Cambrian age (600 ± 150 Ma) whereas the younger granites which are anorogenic and intrusive into the basement are of Jurassic age (150 Ma). The area is underlain by basement complex rocks which are mostly migmatites (fig.1) although exposure of these rocks is limited, they are seen outcropping at the north-eastern and north western part of the study area. The out crops are of low relief when compared with the porphyritic biotite granite and hornblende granite. The contact between the migmatites and the porphyritic biotite granite is gradational with structural conformity. Other rock units include older basalts located around the southern and central parts of the study area. The older basalts are generally small, eroded and partly decomposed remnants. The newer basalts which occur chiefly as flows within the basement especially at the north-eastern and south-eastern parts of the study area are dark coloured, fine grained and composed mainly of plagioclase feldspars, olivine and quartz.

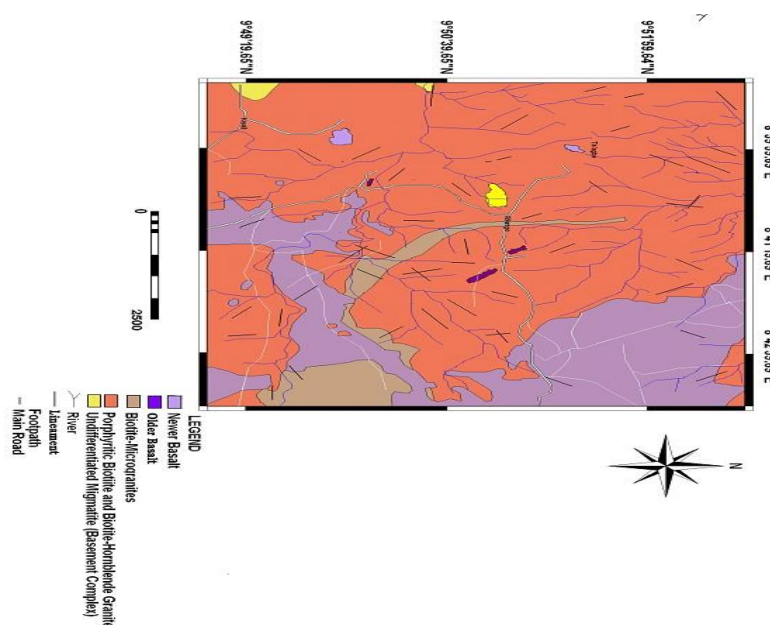


Fig.1: Geologic map of study area

2.0: LITERATURE REVIEW

Krumbein and Grayhill (1965) have distinguished three types of models in geology: (1) scale-models; (2) conceptual models; and (3) mathematical models. Traditionally, geologists have been concerned with scale-models and conceptual models mainly.

Examples of scale-models are the geological map and cross-sections where the spatial variability of attributes is represented at a reduced scale for topographic surface and vertical planes, respectively. Geological processes also can be represented by scale-models. A classic discussion of this subject was given by Hubbert (1937). Conceptual models are mental images of variables and constants. They are statistical or deterministic depending on whether one or more random variables are used in the equation or systems of equations to express uncertainty. Mathematical equations generally can be represented geometrically by curves or surfaces.

The three types of models listed are not mutually exclusive. Scale-models can be based on mathematical criteria and conceptual models may be partly or entirely quantitative. Most mathematical models in geology have some important aspects of uncertainty and for this reason, are statistical. The problem may consist of eliminating the random variations from data so that a deterministic expression is retained representing the relationship between averages for assemblages of attribute rather than between single features. Statistical components or the uncertainty provide a way of expressing a range of different extrapolations for single features, all of which are possible, but with different probabilities of occurrence. This method replaces that of extrapolating a phenomenon with absolute certainty.

Geology differs from physics, chemistry and other sciences in that the possibility of doing controlled experiment is more limited. The observations are restricted to a record of past events, making geology a historical science. Generally, the final product of many interrelated processes is exposed at the surface of the earth in an imperfect manner. These mainly physical-chemical processes seldom reached a state of

equilibrium; most came to a halt before reaching equilibrium at one or more specific points of time.

Basalts mainly consist of two minerals: plagioclase and pyroxene. Barth (1962) discussed the following simplified model. Basalt magma can be defined as a silicate melt that on cooling yields plagioclase and the pyroxene. These may be considered in terms of two main reaction series: (1) the series of plagioclase feldspars going from calcic to sodic composition (Ca→Na); and (2) a series of clinopyroxenes developing from diopside to hypersthene (Mg→Fe). Although this is an oversimplification of pyroxene crystallization relationships, it will be helpful for our discussion to use this combination of the two processes for representing the principal crystallization process of basaltic magma. At specific compositions of magma, the second series can begin with initial precipitation of olivine that, on further cooling is converted into pyroxene.

The relationship between the two processes is shown graphically in Fig.2. There is a boundary OP in the center of this illustration. If the composition of the original basaltic liquid lies to the right of OP, crystallization will start precipitation of only pyroxene. If it is located to the left of the boundary, only plagioclase will be formed. Because of the precipitation of crystals, the magma composition will change describing a path that is directed to the center. Once the line OP is reached, simultaneous precipitation of both mineral phases takes place. A melt of composition O crystallizes by simultaneous formation of pyroxene of composition B and plagioclase of composition Q. As crystallization proceeds, the melt changes from O to P and the two solid phases change from B to R and from Q to A. Fig2 is only approximate and the model which has four constituents cannot be completely represented by it. For a more complex representation, a tetrahedron with four corner points can be used for projection (cf. Barth, 1962). One of the objectives of a model of this type is to classify a given basaltic rock. After the mode of the norm has been calculated for a basalt sample, the result can be projected on a phase diagram and from the position of the point with respect to one or more boundaries such as OP in Fig.2, it can be conducted which solid phase has crystallized first: plagioclase or pyroxene.

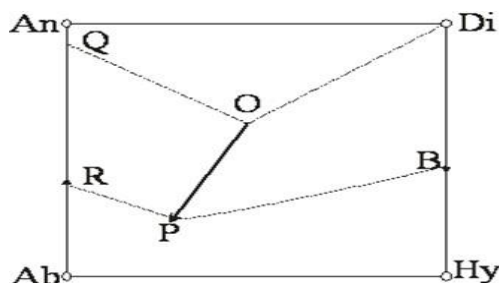


Figure 2: Schematic Presentation of the Crystallization process of Basaltic lavas
Source: From Barth (1962).

Another practical application is as follows. Suppose that basaltic magma occurs in an underground magma chamber which is subject to cooling and where continuous precipitation of crystals takes place in the manner described above. Suppose further that periodically some magma escapes from the chamber to the surface of the earth where it forms basalt lavas. An

assemblage of samples from these lavas may show some regularities in the relationship between components. Ca-rich plagioclase will tend to coexist with Mg-rich pyroxene. If the major oxides for a suite of basalts are correlated to one another, regardless of time deposition, the following signs may occur for the coefficients in the correlation matrix as presented in table 1.

Table 1: Coefficient of Correlation Matrix

	FeO	MgO	CaO	Na ₂ O
FeO	1			
MgO	-	1		
CaO	-	+	1	
Na ₂ O	+	-	-	1

Source: From Barth (1962).

Likewise, if the major oxides are plotted against time, the cooling of an underground magma chamber may be reflected at the surface where CaO and MgO decrease whereas FeO and Na₂O increase with time. In this example, the time of deposition can be entered as a variable by sampling successive flows in a volcanic pile from bottom to top.

Bowen's reaction principle a concept, first propounded in 1928 by Norman Bowen, which explains how mineral can respond to changing equilibrium conditions when a magma is cooled, by either a continuous diffusing – controlled exchange of elements with the magma or discontinuous melting of the material.

The periodic law was developed independently by Dmitri Mendeleev and Lothar Meyer in 1869. Mendeleev created the first periodic table and was shortly followed by Meyer. They both arranged the elements by their mass and proposed that certain properties periodically reoccur. Meyer formed his periodic law based on the atomic volume or molar volume, which is the atomic mass divided by the density in solid form. Mendeleev's table is noteworthy because it exhibits mostly accurate values for atomic mass and it also contains blank spaces for unknown elements.

Goldschmidt (1937) proposed his Classical general rules to explain the distribution of the elements, in which ions of similar size and charge substitute themselves.

Ringwood (1955) proposed the complementary use of the concept of electronegativity in order to understand the distributions of the chemical elements that could not be explained completely with the Goldschmidtian rules, especially when the minerals being investigated had high percentages of covalent bonding.

Nickold (1966) proposed that the three principal factors (ionic size, ionic charge and electronegativity) be

expressed in a single function that would not result in the dichotomous predictions.

Bernard J. Wood (2003), Modifies Goldschmidt rules 2 and 3, that the site has a preferred radius of Ion (r) which enters mostly easily, for ions of the same charge, those which are closest in radius to enter most easily, ions which are larger or smaller are discriminated against.

Fournier and Rowe, (1966), state that silica Geothermometer works because that solubility of the various silica minerals (Quartz, and chalcedony, SiO₂) increase monotonically with temperature.

White (1970) state Na – K Geothermometer takes advantage of the fact that the equilibrium points of certain exchange reactions among various minerals, principally, the feldspar vary with temperature.

Spatial thinking is essential to all spatially dependent sciences, including geography. It comprises the spatial abilities of visualization, orientation and, likely also relation, although contested, (Bednarz & Lee, 2019).

Toramaru, A., and Kichise, T., (2023), proposed a new numerical experiments to study the influence of different cooling rates and classical nucleation theory parameters on the crystal number density measured under constrained conditions in the laboratory experiments.

2.2: A Mathematical Model of Vulcanian Eruption

Explosive volcanism presents a natural hazard on a global scale (Turcotte and Ockendon 1990).

Although extensive qualitative classifications of volcanic eruption have been made (Williams and McBirney 1979), relatively few quantitative studies have been carried out. Explosive volcanic eruptions can generally be divided into

classes (1) vulcanian eruptions are transient and resemble explosion (Selt, Wilson and Nairn 1979) and (2) plinian eruptions are relatively slowly varying in time with a pressure driven jet generating a plume of ash (Wilson 1980).

Although a number of papers have considered the I – D expansion of magma with gases in steady nozzle flows, the purpose of this paper is to formulate and solve the I-D, unsteady wave problem. A number of assumptions will be made to make the problem tractable but once formulated exact solutions will be obtained. Our results may be applicable in a number of other fields.

2.3: Petrologic and Mathematical Modeling of Basaltic Rocks from Venus: A Case for the Presence of Silicic Rocks

One major geological difference between the earth and all other terrestrial bodies (i.e. planets and asteroids) is the presence of evolved silicic igneous rocks (e.g. granite and rhyolite). The continental crust of earth is composed mostly of silicic rocks with small amounts of sedimentary and mafic rocks whereas the oceanic crust is primarily composed of basaltic rocks and about 3% silicic igneous rocks (Dixon and Rutherford 1983; Bonin et al, 2001; Rudnick and Gao 2003). Continental mafic large igneous provinces are temporarily, spatially and structurally contiguous regions of the crust which are predominantly composed of basaltic rocks and cover large (i.e. 71052km) areas (Jerran and Widdowson 2005; Ernest et al., 2005; Byan and Ernest, 2008). Some LIPS are considered to be physical expression of mantle upwelling where not compositionally primitive silicate magmas are transferred to the crust (Ernest and Buchan 2003). The presence of silicic rocks within the LIPS demonstrates that high temperature igneous process will produce compositionally evolved magmas (Mellubo et al., 2008; Xu et al., 2010; Shellnut et al., 2011).

The origin of silicic rocks within oceanic crust and mafic continental LIPS is debated but they may be formed by (1) fractional crystallization of mafic magmas (2) partial melting of mafic rocks (3) partial melting of crystal rocks (4) silicate liquid immiscibility (Bellieni et al., 1986; Shellnut and John, 2010; Xu, et al., 2010; Charlier and Grove, 2012).

2.4: Metrical Matrices of Rock Forming Minerals

The use of matrices to resolve some problems concerning chemical distribution and elemental substitution of chemical elements in rocks becomes very important in this research.

In mathematics, a matrix is a rectangular array of numbers, symbols or expressions, arranged in rows and columns according to Cayley. A., (1821-1895), these numbers are called the elements or entries of the matrix. Matrices have wide applications in engineering, physics, economics, and statistics as well as in various branches of mathematics and even geology.

According to **G.V. GIBBS** the calculation of the d-spacing, the angles between planes and zones, the bond lengths and angles and other important geometric relationships for a mineral can be a tedious task both for the student and the instructor, particularly when completed with the large assortment of trigonometric identities and algebraic formulae that are available (cf. Crystal Geometry (1959), Donnay and Donnay, International Tables for Crystallography, Vol. II, Section 3, The Kynoch Press, 101-158). However, such calculations are straightforward and relatively easy to do when completed with the metrical matrix and the interactive software MATOP. In constructing a model for S, we chose three noncoplanar, coordinate axes denoted X, Y and Z, each radiating from the origin, 0. Next, we place three nonzero vectors denoted a, b and c along X, Y and Z, respectively, likewise radiating from 0. Since the vectors denoted a, b and c along X, Y and Z, respectively, likewise radiating from 0. Since the vectors $D = [a, b, c]$ are nonzero, noncoplanar vectors, they qualify as basis vectors for S. This means that for each vector v in S there exist three real numbers x, y and z such that v can be written as $v = xa + yb + zc$.

3.0: METHODOLOGY

This is Description of the samples in terms of physical and chemical characteristics and how the samples are obtained either from the field or other source. The sources are mainly from the internet, library, textbooks, and Journals. Mathematical mappings, models equations and thematic thermal map, would be used to analyze the minerals and rock samples with respect to thermodynamic principle.

3.1: Rock Thin Section Microscopic Method

Rock samples can be analyzed using petrographic microscope both plane and cross polarized light

3.2: Chemical Analysis

The chemical analysis of basalt and Granite can be analyzed using Inductively Coupled Plasma - Mass Spectrometer (**ICPMS**). **ICPMS** method analyzes the major elements in their percentages and can be converted to their oxides by their conversion factors

3.3: Thematic Map Of Magma

Thematic analysis is a method for identifying, analyzing and reporting patterns (themes) within data (Braun and Clarke, 2006).

According to Kenneth Field (2016), thematic mapping examines the innovative and fascinating alternative ways of making maps of data which you can use in your work, that speak to your truth or which will speak to your truth.

3.3.1: Thematic Thermal Magmatic Contour Map

A Thematic Thermal Magmatic Contour Map is designed purposely for this work to represent the intensity

of “thermal” occurrence within a data set or system (magma). This map is connected to the “Particles of the Universe” and “the Electromagnetic Spectrum” which absorbs or emits energy as a “Photon” as well as Silica polymorphs such as Cristoballite, Tridymite and Quartz as shown in Figure (1). This connection is due to thermal occurrence in the map which represents Magmas, because every object in its highest potential energy at high temperature absorbs certain amounts of energy from the universe, but under its low potential energy at its lowest energy emits the same quantum amount of energy to the universe inform of “LIGHT” called “PHOTON”, This type of map uses colour to represent the thermal intensity of

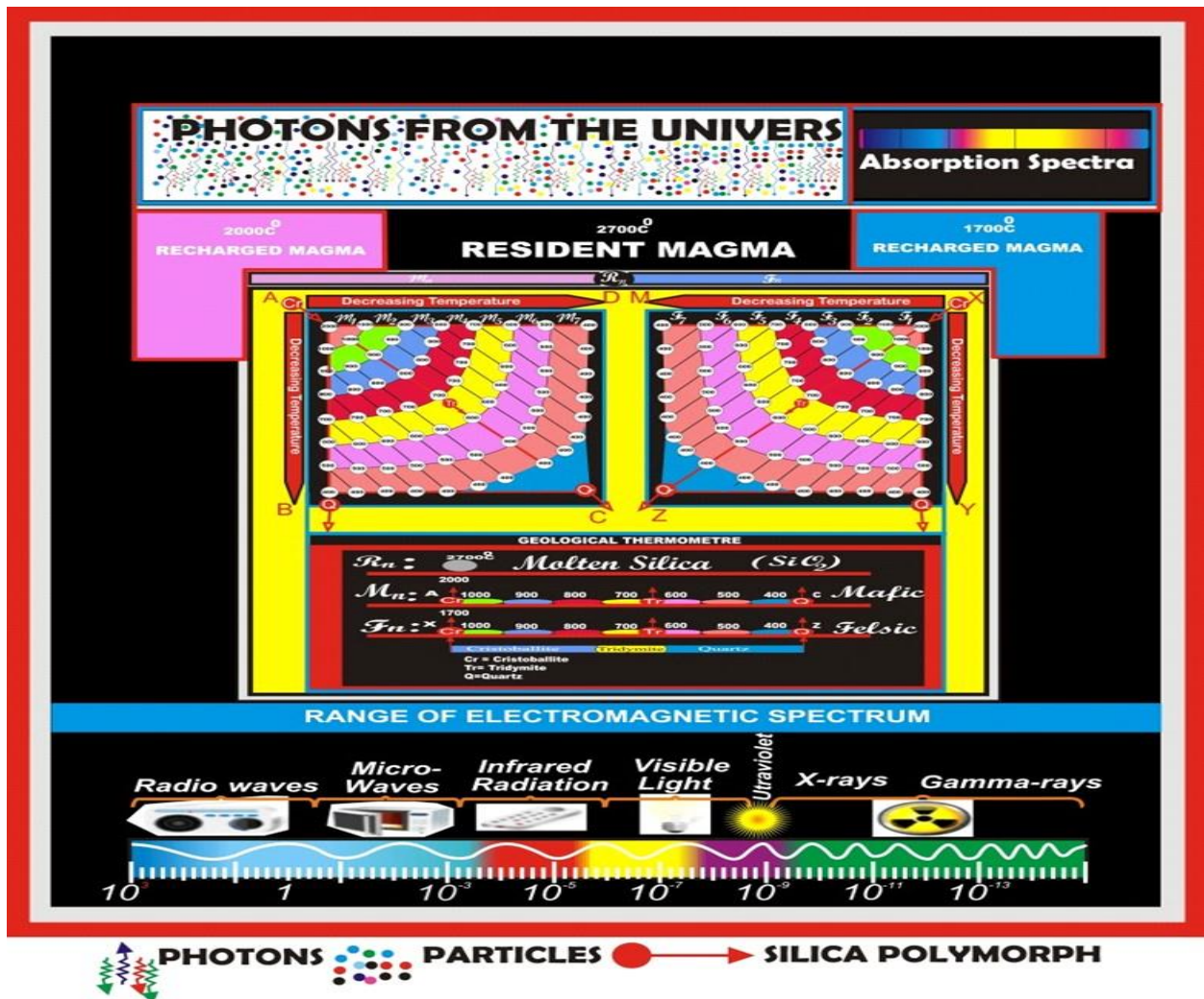


Figure 3: Thematic Thermal Contour Map.

3.4: Mathematical Methods

Mathematical methods are used to resolve the problem of elemental substitution and distribution of chemical elements in rock throughout the time of crystallization from the beginning to the end of crystallization. The mathematical methods include: Matrix method

- a. Geometric Method
- b. Set Notation Method

3.4.1: Metrical Matrix

Boisen, Jr., M.B. and G.V. Gibbs, (1988) used metrical matrix to find the equation of a lattice (crystal) plane and its Miller indices.

$$\text{Where } G = \begin{bmatrix} a^2 & ab \cos \gamma & ac \cos \beta \\ ab \cos \gamma & b^2 & bc \cos \alpha \\ ac \cos \beta & bc \cos \alpha & c^2 \end{bmatrix}$$

Suppose that the unit cell dimensions of a triclinic rhodonite crystal are $a = 10.497\text{\AA}$, $b = 9.797\text{\AA}$, $c = 12.185\text{\AA}$, $\alpha = 103.00^\circ$, $\beta = 108.51^\circ$ and $\gamma = 82.50^\circ$. with this information, the metrical matrix becomes

$$G = \begin{bmatrix} 110.187009 & 13.423197 & -40.606321 \\ 13.423197 & 95.981209 & -26.853857 \\ -40.606321 & -26.853857 & 148.474225 \end{bmatrix}$$

3.4.3: Rough Set Theory

Shaaban M. Shaaban, Sameh Z. Tawfik (2020), rough used set theory to extract volcanic rocks information effectively, a large number of characteristic data must be objectively filtered out. When the best combination of characteristic parameters is achieved, it can be used to identify volcanic rocks precisely. After evaluating many non-linearity computational methods, no further data or previous knowledge were found to be needed for RS theory. The study of geological and volcanic rock information based on RS is a sort of new solution to the

mainly geological high-dimensional complex NP (Nondeterministic Polynomial) problems.

1. Information System
2. Indiscernible Relation
3. Attribute Reduction

3.4.4: Mathematical Model of Volcanic plumes

Joshua Manfred Duley(2014), established a simple mathematical model for a volcanic plume as a mechanism for the transport of suspended particles and their subsequent fallout and the chief focus being a column generated by volcanic events. This explains how the model into the modern understanding of Plinian eruption. Fluid and thermodynamics clouds as a particular phenomenon of Plinian Volcanic eruption.

A simple illustration can be shown in Figure 5 below by assuming and taking thin horizontal slices of plumes with trapezoidal vertical cross-section which, and when rotated about a central axis, are similar to partial cones. In addition to these local shape characteristics, columns produced in plinian eruptions will have what is known as umbrella shape, similar to mushroom clouds seen in atomic detonations. As plume gases begin to lose their kinetic energy, near the apex of the plume, they slow down rapidly causing an accumulation of mass. As more gas rises, it forms lateral displacement of volume, seen as a rapid expansion in the plume generating what is commonly as the umbrella region.

Within the obvious difficulties surrounding measurements on the interior of plume, the velocity profile is typically assumed to be either Gaussian, with the plume travelling faster at its core or, more simply, a top-hat model that is with uniform velocity for a given altitude within the plume and the atmosphere stationary as shown in Figure 4, below. These assumptions, are explored experimentally by solovitz and Mastin (2009) and the approach is used in modeling plumes regularly.



Figure 4: An aerial image of the May 18, 1980, Eruption of the Mt St Helen's, Washington courtesy of the U.S. Geological Survey.

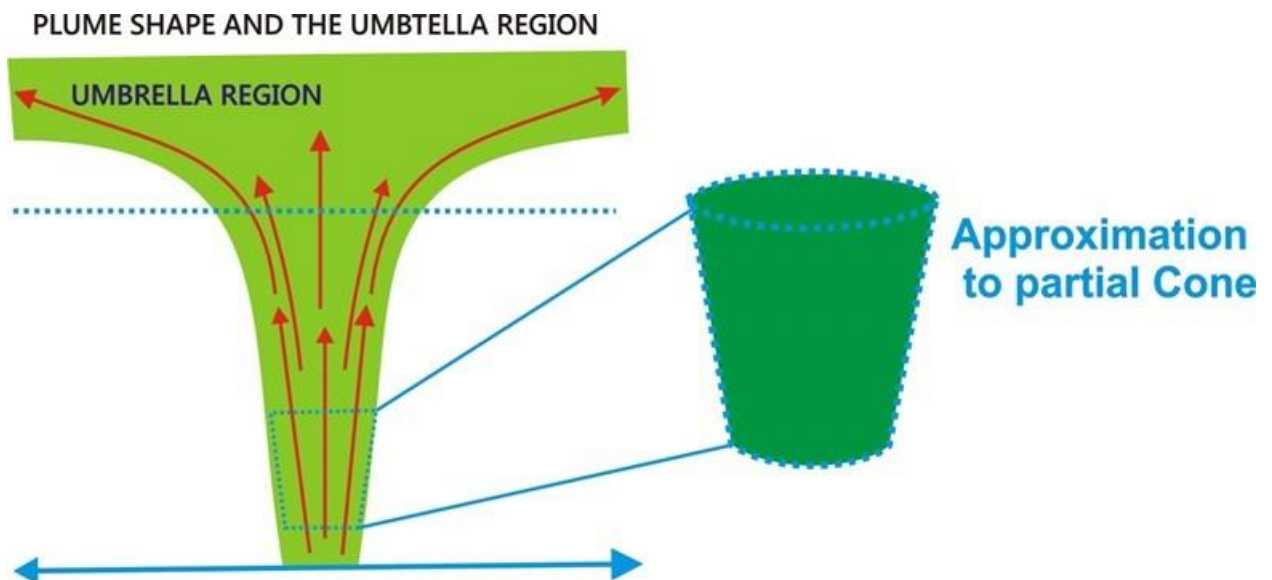


Figure: 5; A Schematic Illustration of the umbrella Region of ash clouds and approximation to a thin slice.

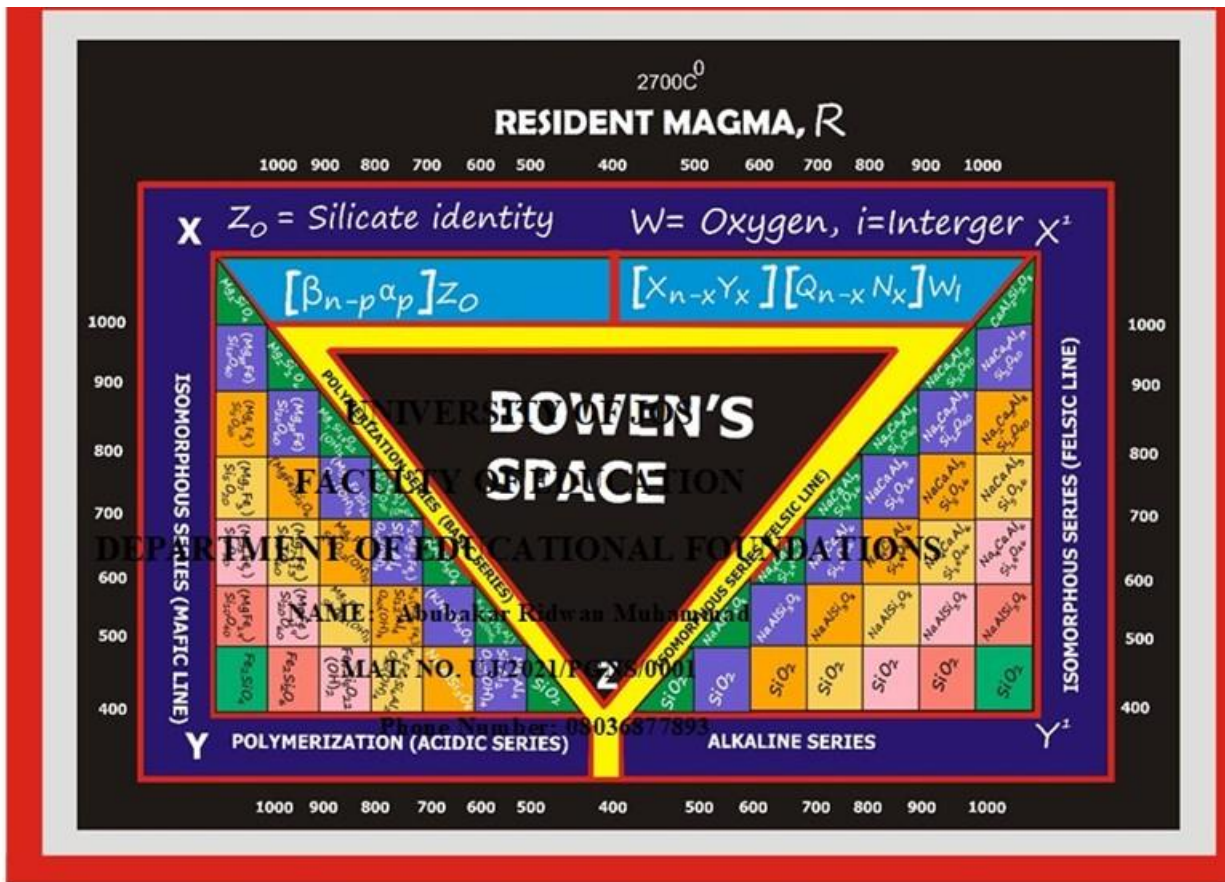


Figure 6: A schematic illustration of the Top-hat and Gaussian velocity models, indicating their relation to the plume boundary at height, $z = Z$

The top hat model can be expressed mathematically as;

$$U_0(r, \phi, Z) = \begin{pmatrix} U_0(r, \phi, z) \\ V_0(r, \phi, z) \\ W_0(r, \phi, z) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ W_0(z) \end{pmatrix}$$

Over the range $0 < r \leq R(Z), 0 < \phi \leq 2\pi, Z \geq Z_e$, where U_0 is the plume velocity, $R(z)$ is the radius of the plume at height z , and Z_e the vent altitude

3.5: Method Adopted In This Research

Given that;

$$\sum_{p=0}^n \binom{n}{p} [\beta - \alpha]_{n+p} (Z_0)_{\delta} = \binom{n}{p} [\beta_{n-p} \alpha_p] (Z_0)_{\delta}$$

Where Z_0 is the silicate identity and

$$\eta = \frac{z^-}{x^+} \text{ (ionic species)}$$

' η ' is the strength of the magma \mathfrak{B} , which is the ratio of the silicate radical z^- to that of cation x^+ under electrolytic condition and 'p' is the recipient cation which depends on ' η '.

$\binom{n}{p}$ is the coefficient of rock forming minerals and it determines the number of outcomes of each mineral in the melt $\mathfrak{R}(Z_0)_{\delta}$.

3.5.1: Expected Mathematical connection between Bownens and Goldschmidt concept using Binomial Expansion

Therefore for;

$$\sum_{p=0}^n \binom{n}{p} [\beta - \alpha]_{n+p} (Z_0)_{\delta} = \binom{n}{p} [\beta_{n-p} \alpha_p] (Z_0)_{\delta}$$

$$\sum_{p=0}^n \binom{n}{p} [\beta - \alpha]_{n+p} (Z_0)_{\delta} = \binom{n}{p} [\beta_{n-p} \alpha_0 (Z_0)_{\delta} - \binom{n}{p} [\beta_{n-p} \alpha_1 (Z_0)_{\delta} + \binom{n}{p} [\beta_{n-p} \alpha_2 (Z_0)_{\delta}$$

Mathematically, in complex analysis;
 $i^2 = -1$

$$\sum_{p=0}^n \binom{n}{p} [\alpha + \beta]_{n+p} (Z_o)_{\delta} = [\beta_n] (Z_o)_{\delta} + i \{ [\beta_{n-1} \alpha_p] (Z_o)_{\delta} + [\alpha_p] (Z_o)_{\delta} \}$$

'i' is a complex number in silicate Magma.

$$({}^n C_p) = \binom{n}{p}$$

$$\binom{n}{p} = \frac{n!}{(n-p)! p!} \text{ and } 0! = 1$$

$\delta = o, p, a, m$ and f

Where, o, p, a. and m are olivine, pyroxene, amphibole, mica and feldspar

3.6: Anhydrous Melt,

Magma with no water content at high temperature, silica deficient and low viscosity.

3.6.1: Expected Mathematical expression for Mafic Olivine series:

$$\sum_{p=0}^n \binom{n}{p} [\alpha + i\beta]_{n+p} (Z_o)_{\delta} = 1[\beta_n] (Z_o)_{\delta} - 2i[\beta_{n-1} \alpha_1] (Z_o)_{\delta} + 1[\alpha_2] (Z_o)_{\delta}$$

Given that, $\beta = \text{Magnesium}$
 $\alpha = \text{Iron}$
 $Y = \text{Calcium, Chromium, Nickel or Manganese}$

$$(Z_o)_{\delta} = Si O_4$$

From equation (13), we can rewrite it as;

$$\sum_{p=0}^n \binom{n}{p} [\alpha + i\beta]_2 Si O_4 = 1 Mg_2 SiO_4 + i \{ 2[MgFe] SiO_4 \} + 1 Fe_2 Si O_4$$

$$[Mg + iFe]_2 Si O_4 = Mg_2 Si O_4 + i[Mg Fe] Si O_4 + i[MgFe] SiO_4 + Fe_2 Si O_4$$

$$\sum_{p=0}^n \binom{n}{p} [Mg + iFe]_{n+p} Si O_4 = \text{Forsterite} + i[\text{Hyalosiderite}] + i[\text{Hortonolite}] + \text{Fayalite}$$

Mg_{2-p}Fe_pSiO₄ is an '**Olivine series**' formula that can be used to calculate all olivine minerals from the melt \mathfrak{M} , where p is an integer and ranges from 0 to 2 in olivine crystals. At p equals to zero, 100% Forsterite (Fo) crystallizes with chemical formula of Mg₂SiO₄ and at p equals to 2, Forsterite (fo) disappears and 100% of pure Fayalite (Fa) crystallizes with chemical formula of Fe₂SiO_{4(s)}. Therefore forsterite, Hyalosiderite, Hortonolite

and fayalite are the primary olivines in the mafic olivine series.

3.6.2: Expected Mathematical Expression for Mafic Pyroxene series:

$$\sum_{p=0}^n \binom{n}{p} [\beta - p\alpha] (Z_o)_{\bar{\delta}} [\beta + \alpha]_{2+p} Si_2 O_6 = 1 Mg_2 Si_2 O_6 + i \{ 2[MgFe] Si_2 O_6 \} + Fe_2 Si_2 O_6$$

$$\sum_{p=0}^n \binom{n}{p} [\beta + \alpha]_{n+p} Si_2 O_6 = Mg_2 Si_2 O_6 + i \{ 2[MgFe] Si_2 O_6 \} + i[MgFe] Si_2 O_6 + Fe_2 Si_2 O_6$$

Pyroxene = Enstatite +Magnesio-hypersthene + Ferro-hypersthene + Ferrosilite

Mg_{2-p}Fe_pSi₂O₆ is a pyroxene series, where p range from 0 to 2 in pyroxene. At p equals to zero, 100% Enstatite (En) crystallizes with chemical formula of Mg₂Si₂O₆ and at p equals to 2, Enstatite disappears and 100% of pure Ferrosilite crystallizes with chemical formula of Fe₂Si₂O₆

Therefore Enstaite, Hypersthene, Eulite and ferrosilite are the primary pyroxenes in the mafic pyroxene series.

3.7: HYDROUS MELT

Magma with water content at low temperature, silica saturation and high viscosity

3.7.1: Expected Mathematical Expression for Mafic Amphibole

$$[\beta_{7-p} \alpha_p] (Z_o)_{\bar{a}2} = Si_4 O_{11} (OH)^{-7}$$

$$(Z_o)_{\bar{a}2} = Si_8 O_{22} (OH)_2$$

$$\sum_{p=0}^n \binom{n}{p} [\beta + \alpha]_7 (Z_o)_{\bar{a}2} = 1[\beta_7] (Z_o)_{\bar{a}2} + 7i[\beta_6 \alpha_1] (Z_o)_{\bar{a}2} + 21[\beta_5 \alpha_2] (Z_o)_{\bar{a}2} + 30i[\beta_4 \alpha_3] (Z_o)_{\bar{a}2} + 30[\beta_3 \alpha_4] (Z_o)_{\bar{a}2} + 21i[\beta_2 \alpha_5] (Z_o)_{\bar{a}2} + 7[\beta_1 \alpha_6] (Z_o)_{\bar{a}2} + 1i[\alpha_6] (Z_o)_{\bar{a}2}$$

$$\sum_{p=0}^n \binom{n}{p} [Mg + Fe]_7 Si_8 O_{22} (OH)_2 = 1[Mg_7] Si_8 O_{22} (OH)_2 + 7i[Mg_6 Fe_1] Si_8 O_{22} (OH)_2 + 21[Mg_5 Fe_2] Si_8 O_{22} (OH)_2 + 30i[Mg_4 Fe_3] Si_8 O_{22} (OH)_2 + 30[Mg_3 Fe_4] Si_8 O_{22} (OH)_2 + 21i[Mg_2 Fe_5] Si_8 O_{22} (OH)_2 + 7[Mg Fe_6] Si_8 O_{22} (OH)_2 + 1i[Fe_6] Si_8 O_{22} (OH)_2$$

Then;

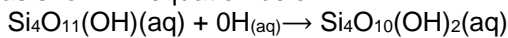
$$[\beta_{7-p} \alpha_p] (Z_o)_{\bar{a}2} = [(Mg_{7-p} Fe_p) (Si_8 O_{22} (OH)_2)]$$

$[(Mg_{7-p}Fe_p)(Si_8O_{22}(OH)_2)]$ is a chemical formula for an Amphibole series, when p ranges from 0 to 7. At p equals to zero, 100% of Kupfferite (Ku) with chemical formula of $Mg_7Si_8O_{22}(OH)_2$ crystallizes. At p equals 7, Kupfferite disappears, and 100% pure Grunerite (Gr) crystallizes with chemical formula of $Fe_7Si_8O_{22}(OH)_2$

3.7.2: Expected Mathematical Expression for Mafic mica series:

$$[\beta_{6-p} \alpha_p](Z_0)_{\bar{m}2}$$

The crystal of amphibole interacts with the magma, so that the $Si_4O_{11}(OH)$ of amphibole links to cation is being hydrolyzed in the presence of excess water in the magma as shown in equation below



$$(Z_0)_{\bar{a}2} = Si_4O_{20}(OH)_4$$

$$\sum_p \binom{6}{0} [Mg + iFe]_6 Si_4O_{20}(OH)_4 =$$

$$1[Mg_6]Si_4O_{20}(OH)_4 + 6i[Mg_5Fe_1]Si_4O_{20}(OH)_4 +$$

$$15[Mg_4Fe_2]Si_4O_{20}(OH)_4 + 20i[Mg_3Fe_3]Si_4O_{20}(OH)_4 +$$

$$15[Mg_2Fe_4]Si_4O_{20}(OH)_4 + 6i[Mg_1Fe_5]Si_4O_{20}(OH)_4 +$$

$$1[Fe_6]Si_4O_{20}(OH)_4$$

Mafic Mica = Phlogopite + i[Magnesio-biotite]+ Biotite +i[Ferro-biotite]+ Lepidomelane

$$[\beta_{6-p}\alpha_p](Z_0)_{\bar{m}2} = [Mg_{6-p}Fe_p][Si_8O_{20}(OH)_4]$$

$[Mg_{6-p}Fe_p][Si_8O_{20}(OH)_4]$ is a biotite series, as p ranges from 0 to 6. At 'p' equals to zero, 100% of phlogopite (Ph) crystallizes, and at 'p' equals to 7, Phlogopite (Ph) disappears and 100% pure Lepidomelane (Lp) crystallizes with a chemical formula of $K_2Fe_6Al_2Si_6O_{20}(OH)_4$.

3.8: Expected Mathematical Expression for Plagioclase series:

$$(Y_{m-p}X_p)(Q_{x-y}N_y)W_i$$

$$(Y + X)_{x+y} (Q + N)_{m+p} W_i = (Y_{m-p}X_p)(Q_{x-y}N_y)W_i$$

$$[X_{m-p}Y_p]Z_i = [X_{1-p}Y_p](Z_0)_{\bar{f}2}$$

$$= [X_{m-p}Y_p](Z_0)_{\bar{f}2}$$

$$(Z_0)_{\bar{f}} = Si_4O_4$$

$$(Z_0)_{\bar{f}2} = Si_4O_8$$

Y – Position = Y + X

$$[X_{m-p}Y_p](Z_0)_{\bar{f}2} = [X_{1-p}Y_p] [Q_{4-y}N_y] W_i$$

For p = 1 and y=1

$$[X_{1-p}Y_p] [Q_{4-y}N_y] W_i = [X_{1-1}Y_1] [Q_{4-1}N_1] W_i$$

$$[X_{1-p}Y_p] [Q_{4-y}N_y] W_i = [Y_1] [Q_3N_1] W_i$$

$$Y_m [Q + N]_{x+y} + [x]_p(Q + N)_{x+y} =$$

$$[X_{1-p}Y_p] [Q_{4-y}N_y] W_i$$

$$Y_m [Q + N]_{x+y} + [x]_p(Q + N)_{x+y} \text{ is called}$$

“PLAGIOCLASE SERIES”

$$Y_m [Q + N]_{x+y} + [x]_p(Q + N)_{x+y} = CaAl_2Si_2O_8 + NaAlSi_3O_8$$

The Equation, $Y_m [Q + N]_{x+y} + [x]_p(Q + N)_{x+y}$ is used to solve the problem involving couple substitution in felsic rock.

3.9: Application of Metrical Matrices to Rock Forming Minerals

Therefore the above equations for both the Mafic and Felsic minerals can be set into matrices to form rocks of desired compositions.

3.9.1: Metrical Matrix for Mafic Olivine and Feldspars;

$$[Y_1(Q_2N_2)]_08 + X_1(Q_3N)08 = [(1, 0), [2, 0]] + [(0, 1), [3, 1)]$$

$$[\beta_2](Z_0)_{\bar{o}} + [\alpha_2](Z_0)_{\bar{o}} = [2, 0] + [0, 2]$$

$$\begin{bmatrix} \text{Anorthite} \\ \text{Forsterite} \end{bmatrix} + \begin{bmatrix} \text{Albite} \\ \text{Fayalite} \end{bmatrix} = \begin{bmatrix} \text{Anorthoclase} \\ \text{Olivine} \end{bmatrix}$$

3.9.2: Metrical Matrix for Mafic Pyroxene and Feldspar

$$\begin{bmatrix} Ca_1[Al_2Si_2]08 \\ [Mg_2]Si_2O_6 \end{bmatrix} + \begin{bmatrix} Na_1[AlSi_3]08 \\ [Fe_2]Si_2O_6 \end{bmatrix} =$$

$$\begin{bmatrix} (Ca_1Na_1)Al_3Si_5]0_{16} \\ [MgFe]Si_2O_6 \end{bmatrix}$$

$$\begin{bmatrix} \text{Anorthite} \\ \text{Enstatite} \end{bmatrix} + \begin{bmatrix} \text{Albite} \\ \text{Ferrosillite} \end{bmatrix} = \begin{bmatrix} \text{Anorthoclase} \\ \text{Pyroxene} \end{bmatrix}$$

3.9.3: Metrical Matrix for Mafic Amphibole and Feldspar

$$[\beta_{7-p}\alpha_p](Z_0)_{\bar{a}}, \text{ at } [7, 0] = [\beta_7](Z_0)_{\bar{a}} \dots (127)$$

$$[\beta_{2-p}\alpha_p](Z_0)_{\bar{a}}, \text{ at } [0, 7] = [\alpha_2](Z_0)_{\bar{a}} \dots (128),$$

$$\begin{bmatrix} Ca_1[Al_2Si_2]08 \\ [Mg_7]Si_8O_{22}(OH)_2 \end{bmatrix} + \begin{bmatrix} Na_1[AlSi_3]08 \\ [Fe_2]Si_8O_{22}(OH)_2 \end{bmatrix} =$$

$$\begin{bmatrix} [(Ca_1Na_1)(Q_5N_3)]0_{16} \\ [MgFe]_7Si_{16}O_{44}(OH)_4 \end{bmatrix}$$

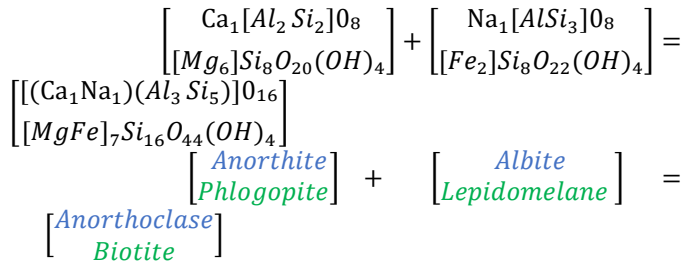
$$\begin{bmatrix} \text{Anorthite} \\ \text{Kupfferite} \end{bmatrix} + \begin{bmatrix} \text{Albite} \\ \text{Grunerite} \end{bmatrix} =$$

$$\begin{bmatrix} \text{Anorthoclase} \\ \text{Anthophyllite} \end{bmatrix}$$

3.9.4: Metrical Matrix for black Mica and Feldspar

$$[\beta_{6-p}\alpha_p](Z_0)_{\bar{m}}, \text{ at } [6, 0] = [\beta_6] (Z_0)_{\bar{m}} \dots (134)$$

$$[\beta_{2-p}\alpha_p](Z_0)_{\bar{m}}, \text{ at } [0, 6] = [\alpha_6](Z_0)_{\bar{m}} \dots (135),$$



3.10: SET NOTATION

Set notation is a mathematical parameter used in this research to partition minor elements of small concentrations in part per million (ppm), such as, transition elements, and Rare Earth Elements according to their compatibility and incompatibility in the magma. The compatibility and the incompatibility, of these minor elements depend on the ratio of the concentration of minor elements in the solid (C_s) to the concentration of the minor elements in liquid C_1 in the magma \mathfrak{M} , during crystallization. The ratio of the concentration of the concentration of minor elements in solid (C_s) to that of the minor elements in Liquid (C_2) is constant (K_n). Therefore, this can be mathematically expressed as

$$K_n = \frac{C_s}{C_L}$$

Where; C_s = concentration in solid

C_L = Concentration in Liquid

K_n = Constant.

Therefore, if the ratio of the concentration of trace elements or REE in solid (C_s) to that of the liquid is greater than one, $K_n > 1$, then the trace or REE is compatible, and can be partitioned into the solid phase. In this case, for compatible trace or REE, k_n such that;

$$\frac{C_s}{C_L} > 1$$

If the ratio of concentration of trace elements, transition or REE in solid (C_s) to that of the Liquid (C_L) is less than one, $k_n < 1$ then the minor elements is incompatible and can be partitioned along the liquid phase in this case, for incompatible minor elements, $K_n < 1$, such that

$$\frac{C_s}{C_L} < 1$$

It pertinent to note that, those minor elements that partitioned along the fluid phase, such as hydrous phase or gaseous phase are mostly High Field strength Elements (HFSEs), because they have small ionic radius and large ionic charge. They from the major gemstones and ore deposit of the world. They are also called small ion Lithophile elements.

The mathematical equations above can be represented using set Notation

$$K_d = [K_d \text{ mineral (1 m the rock)}] + [\mathfrak{M}_{inz}] + [\mathfrak{M}_{in-3}]$$

$$2\% \text{py}, 10\% \text{mag}, 40\% \text{plag}, 30\% \text{Am}$$

$$K_d = (0.2 \times 1.76) + (0.1 \times 0.53) + (0.4 \times 0.061) + 0.3 \times 2.99 = 1.32$$

4.0: EXPECTED OUTCOMES

4.1: BOWEN'S MODEL

Bowen's (1928) uses temperature under isobaric condition to explain sequences of crystallization in which all silicate minerals are formed from the melt as temperature drops from the beginning to the end of crystallization. From this, Bowen's propounded his two reactions series which indicate discontinuous and continuous reactions series where discontinuous reaction series represents mafic minerals and continuous reaction series represents felsic minerals as shown Figure (7) below;

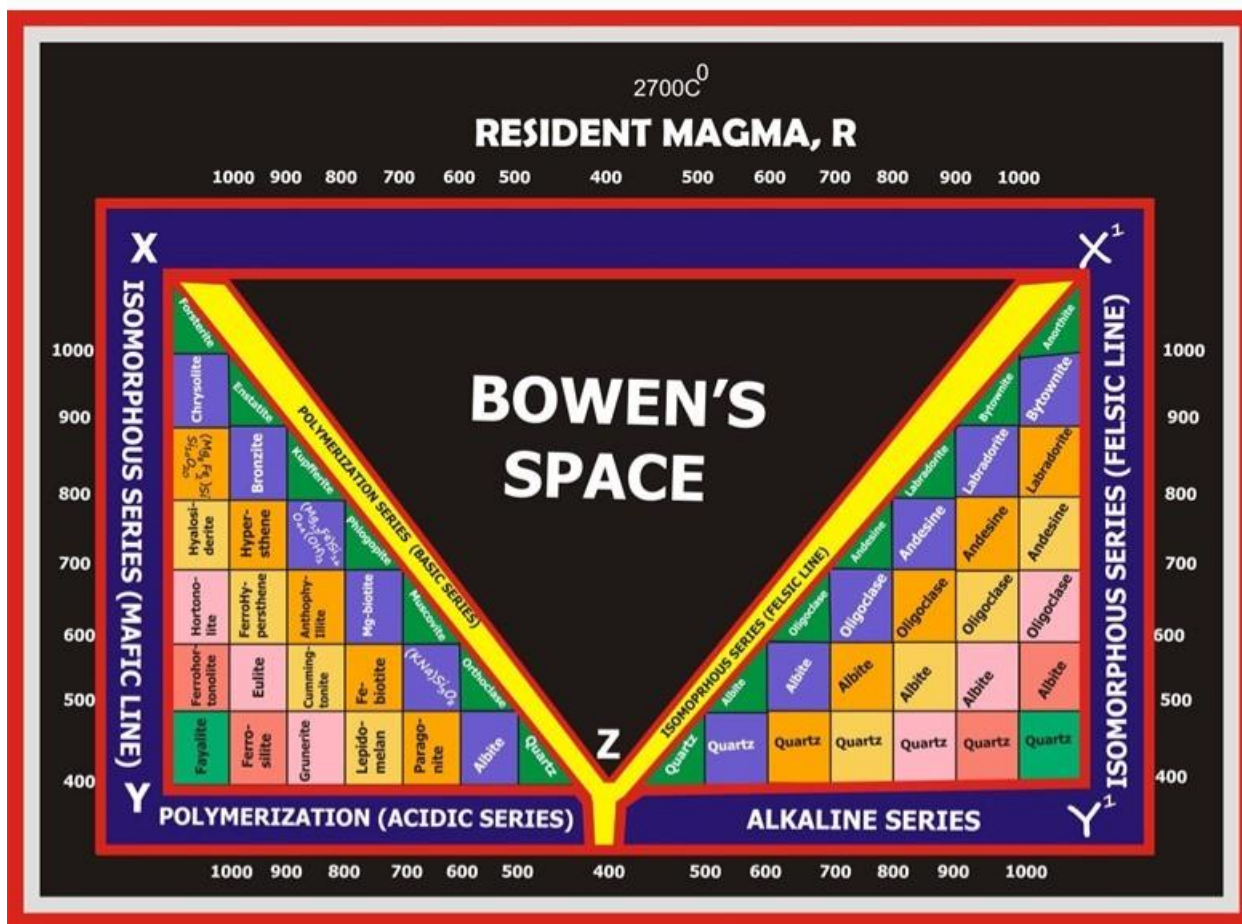


Figure (7): Bowen's Reaction Series

4.2: Bowen's And Goldschmidt Combined Model

This research uses mathematical equations under thermodynamic change to explain sequences of crystallization from the beginning to the end of crystallization and from this, two reactions series would be propounded which complements **Bowen's reactions series**. These two reactions series are **Polymerization** and **Isomorphous** reactions series as shown in Figure (8) below.

Bowen (1928) utilized temperature and time to establish relationships among essential rock-forming minerals, while Goldschmidt (1937) used ionic charge and size to explain ionic substitutions in crystals (Diadochi) as shown in Figure (8) below.

1. Under mathematical context, a significant documented mathematical framework complementing the explanation of Bowen's reaction series, which encompasses the **ISOMORPHOUS** (solid solution) and

POLYMERIZATION (fractional crystallization) reaction series would be propounded using the model as shown in the Figure (8) below.

2. Both Bowen and Goldschmidt showed a mathematical connection in their rules, proffering a complete mathematical explanation of rock-forming minerals and their crystallization processes as shown in the combined model below

3. Specifically, this research would present a coordinate geometric relationship between the two reaction series (ISOMORPHOUS and POLYMERIZATION) which would bridge a notable gap between Bowen's and Goldschmidt's rules to their comprehensive understanding as shown in Figure (8). This research would achieve a holistic understanding that bridge the gap between Bowen's and Goldschmidt's rules with a mathematical foundation to comprehensively explain the formation of rock-forming minerals and their reactions as shown in Figure (8).

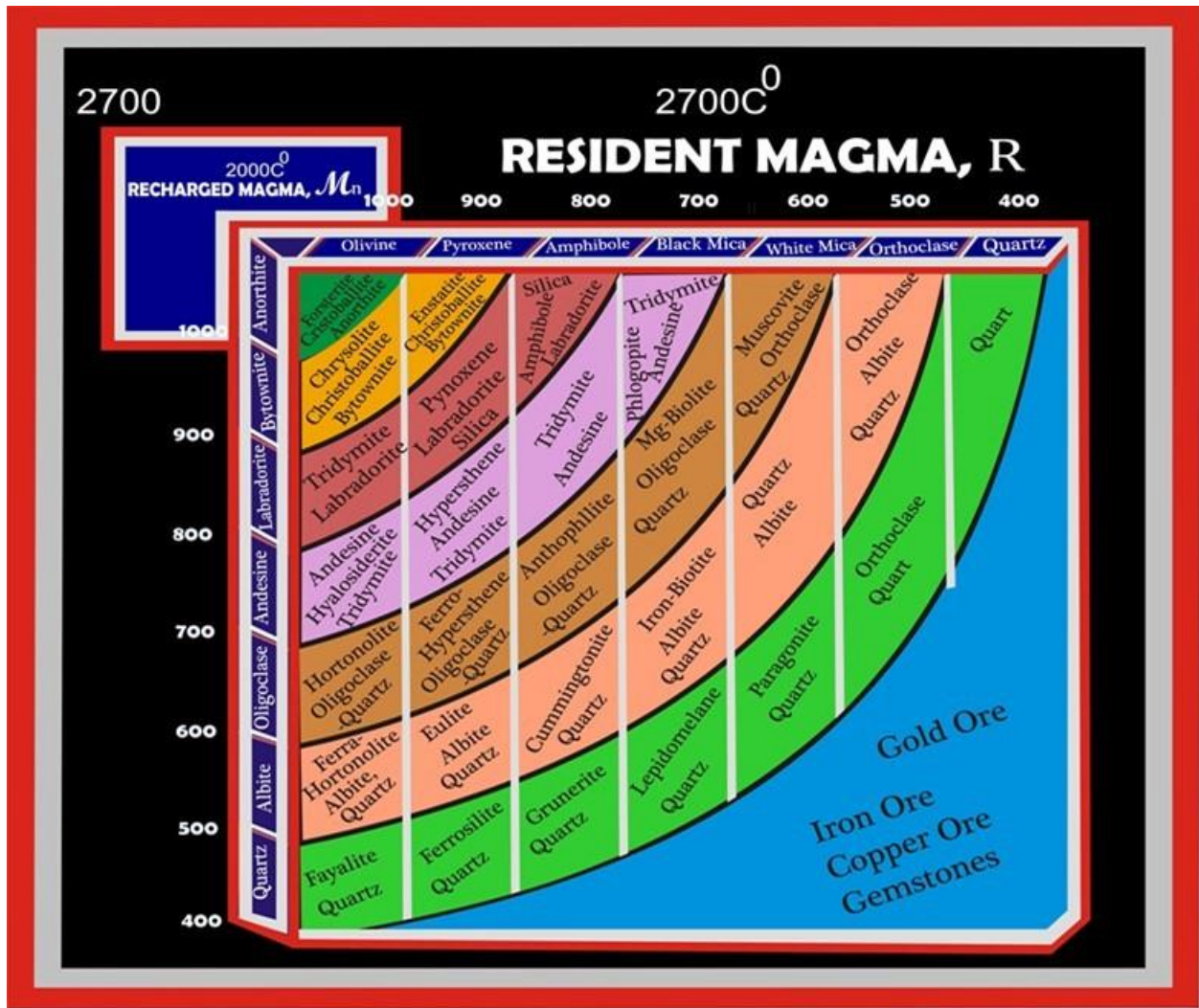


Figure 8: A Model representing Bowen's and Goldschmidt combined Concept

5.0: SUMMARY AND CONCLUSION

5.1: Summary

During the course of this research, the following summaries would be made.

1. Z_0 is the nucleation point of the recharged magma under thermodynamic condition and it is the silicate identity in the magma which is the building block of silicate minerals.
2. $f_n(Z_0) = [\beta + \alpha]_{n+p} Z_0$ is the formula to calculate Mafic minerals from the magma under thermodynamic change
3. $(Y + X)_{m+p} (Q + N)_{y+x} W_x = (Y_{m-p} X_p) (Q_{x-y} N_y) W_i$ is the formula to calculate for complex minerals under thermodynamic change, where

$(Y + X)_{m+p}$ is the lattice of ions of different sizes and charges and $(Q + N)_{y+x} W_x$ is another lattice of ions of different sizes and charges in the same mineral to maintain electrical neutrality in the minerals

5.2: Conclusion

Using Bowen's and Goldschmidt concepts, $f_n(Z_0) = [\beta + \alpha]_{n+p} Z_0$ for Mafic minerals and $(Y + X)_{m+p} (\beta + \alpha)_{n+p} (Q + N)_{y+x} W_i = (Y_{m-p} X_p) (\beta_{n-p} \alpha_p) (Q_{y-p} N_p) W_i$ for felsic and complex minerals were successfully used in this research to calculate the numerical values of minerals in a complex plane of magma from the beginning to the end of crystallization under thermodynamic change and these formulas are valid for all silicate rock forming minerals and the reaction processes are **Polymerization** and

Isomorphous reactions throughout the time of crystallization.

5.3: Contribution to Scientific Knowledge

Bowen (1928) utilized temperature and time to establish relationships among essential rock-forming minerals, while Goldschmidt (1937) used ionic charge and size to explain ionic substitutions in crystals (Diadochi) and

1. Under mathematical context, a significant documented mathematical framework complementing the explanation of Bowen's reaction series, which encompasses the ISOMORPHOUS (solid solution) and POLYMERIZATION (fractional crystallization) reaction series has been propounded using the model.

2. Both Bowen and Goldschmidt showed a mathematical connection in their rules, proffering a complete mathematical explanation of rock-forming minerals and their crystallization processes as shown in the combined model

3. Specifically, this research presented a coordinate geometric relationship between the two reaction series (ISOMORPHOUS and POLYMERIZATION) which bridged a notable gap between Bowen's and Goldschmidt's rules to their comprehensive understanding.

4. This research achieved a holistic understanding that bridged the gap between Bowen's and Goldschmidt's rules with a mathematical foundation to comprehensively explain the formation of rock-forming minerals and their reactions.

6.0 work plan form January 2023-January, 2026.

Task	Goal/Objective	Start date	End date	Duration
Course work	To build and develop research capacity.	May 2023	April 2024	11 Months
Proposal presentation	To explain plans to go about completing the project.	April 2024		
Search and gathering Literatures, review of literatures, acquisition of maps.	To identify similar work done within the area and gaps that demand further investigation. Hence situate current research within the wider discipline	May 2024	July 2025	13 Months
Reconnaissance survey	To check the actual conditions on the ground and note any discrepancies in the maps. To familiarize with community leaders.	August 2024	August 2024	1 month
Phase 1: Field work (Geological mapping, Geological survey, Geophysical survey, Soil sampling, laboratory work, Data and results interpretation)	To confirm existing geology, obtain sub-surface information and obtain disturbed and undisturbed samples. To analyze and data interpret data using appropriate software and standards.	September 2025	March 2026	6 months
Progress report	To present draft of work done so far, receive feedback for further work.	December 2025	December 2025	
Phase 11: (Field work (Geological mapping,, Rock sampling, Laboratory work, Data and results interpretation)	To confirm existing geology, obtain sub-surface information and obtain disturbed and undisturbed samples. To analyze and data interpret data using appropriate software and standards.	April 2026	September 2026	5 months
Progress report	To provide an update on ongoing work and receive feedback	October 2026	October 2026	
Report Writing	To write findings and conclusions in order to produce a final dissertation.	October 2027	January 2027	3 months
Oral Exams (Viva voce)	To defend thesis before appointed examiners(s)	January 2027	January 2027	

7.0. Budget

	Expenditure	Amount (N)
Geological Mapping	b) Rentage of GPS for 2 weeks (N 10, 000 per day)	140,000.00
	a) Data acquisition by 2 technicians for two weeks (N 5, 000 per day)	100,000.00
	(c) Data processing	100,000.00
	(d) Software for data interpretation	450,000.00
Geological Mapping	(a) Equipment hire for sampling for 2 weeks	800,000.00
	(b) Sample bags -100 at N300 per bag	20,000.00
	(c) Standards	100,000.00
	(d) Petrographic analysis	120,000.00
	(e) Mineralogical analysis	200,000.00
	(f) Chemical analysis	100,000.00
	(g) Heating	100,000.00
	(h) Cooling	100,000.00
	(i) Other tests	100,000.00
	(j) Labour for field assistants	100,000.00
Others	Maps	100,000.00
	Typing and printing	80,000.00
	Travels (local international) and accommodation	1,000,000.00
Dissemination of research findings	Publications and conferences	500,000.00
Miscellaneous	Indirect costs	100,000.00
Total		3,310,000.00

REFERENCE

- Andrew, B.J., and Befuse, K, S, (2020). Supersaturation and growth of plagioclase: A numerical Model of decompression induced plagioclase, contribution to mineralogy and petrology, 175(3), 1-20.
- Atkinson, K.E. (1989) An Introduction to Numerical Analysis, Second Edition, John Wiley and Sons.
- Balogun Ometere Deborah, Oluwafemi Israel Oluwarotimi and Famakinwa Olamigoke Emmanuel (2024) Review of Adams-Bashforth method for numerical solution of first order ordinary differential equations. Global Journal of Engineering and Technology Advances, 2024, 19(01), 037–061 DOI: <https://doi.org/10.30574/gjeta.2024.19.1.0056>
- BLOOM (H.) and BOCKRIS (J. O'M), 1959. Molten electrolytes. Chap. 3 in Modern aspects of electrochemistry. No. 2 (ed. J. O'M. Bockris). London, Butter- worth's.
- Bowen N. L. and Schairer, J. F., (1935). The system MgO – Fe O- SiO₂, Amer. J. Sci., 29 (1935), 151.
- Bowen N. L., (1928). The later stages of the Evolution of the Igneous Rocks, Journal of Geology xxiii. Supplement, 91 pp.
- Bowen, N. L. (1914). The system diopside fosterite, silica, Amer. J. Soci., 38. 209.
- Bowen, N. L., (1935). Ferrosilite as natural mineral, Amer. J. Sci., 29, 151.
- Burps, R. G., and Fyfe, W. S., 1966, Distribution of elements in geological processes, *Chem. Geol.*, 1, 49-56. *BumS*,
- R. G., and Fyfe, W. S., 1967, Trace element distribution rules and their significance, *Chem. Geol.*, 2, 89-104. Burns, R.
- G., and Fyfe, W. S., 1967, Crystal-field theory and the geochemistry of transition elements, in *Researches in Geochemistry*, Vol. 2, (Abelson, ed.), Wiley, New ^ York, pp. 259-285.
- Daly, R. A. (1933). Igneous Rocks and their origin, New

158. Glob. Educ. Res. J.

York, 1914. *Igneous Rocks and the Depth of the Earth*, New York.

Daly, R. A. (1933). *Igneous Rocks and their origin*, New York, 1914. *Igneous Rocks and the Depth of the Earth*, New York.

Deer W. A. Howie R. A. Zussman, J. (1963). *Rock – forming minerals in five volumes*, London; vol. 1, 1962; vol. 2, 1963; Vol. 3, vol. 4, 1962. 5, 1962.

Denbigh, K.G., and J.C.R Turner., (1971). *Chemical Reactor Theory – An Introduction (Second Ed.)*. Cambridge, UK: Cambridge Univ. Press (1971).

Fournier, R. O., and Potter, R.W. 11, (1982a): A revised and expand silica (quartz) geothermometer. *Geoth. Res. Council Bull.*, 11 – 10, 3 – 12.

Fournier, R.O., (1979): A revised equation for the Nalk geothermometer. *Geothermal Resources Council Transactions*. Volume 3, 221 – 224.

Fournier, R.O., and Potter R. W. 11, (1979): Magnesium correction to Na-K-Ca geothermometer – *Geochim Cosmochim. Acta*, 43,