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Full Length Research Paper

Natural Convection inside a Discretely Heated Enclosure

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A two-dimensional steady and laminar incompressible flow in a corrugated enclosure is analyzed numerically. Two types of corrugation (vee and sinusoidal) on vertical walls of the enclosure are considered. In this analysis, two vertical corrugated walls are maintained at a constant low temperature, a constant heat flux source whose length is 40% of the total length of the enclosure is discretely embedded at the bottom wall, the non-heated part of the bottom wall and the top wall are considered adiabatic. The pressure velocity form of the Navier-Stokes equations and energy equation are used to represent the mass, momentum and energy conservations of the fluid medium in the enclosure. The Galerkin finite element method has been used to see the effect of corrugation geometry on heat transfer for different Grashof numbers. The average Nusselt number at the heat source surface for different corrugated enclosures are compared with each other. Results are presented in the form of streamline and isotherm plots.

Keywords: Natural convection, corrugation amplitude, penalty finite element, Nusselt number.

INTRODUCTION

Natural convection results when there is a fluid density gradient in a system with a density-based body force such as the gravitational force. It has been studied extensively, both experimentally and numerically, because of its various applications in engineering, such as thermal control in electronic equipment, nuclear reactors, solar collectors, and chemical vapor deposition reactors etc. Heat transfer by natural convection depends in the convection currents developed by thermal expansion of the fluid particles. Further, the development of the flow is influenced by the shape of the heat transfer surfaces. Therefore, the investigation of thermal and fluid flow behaviors for different shapes of the heat transfer surfaces is necessary to ensure the efficient performance of the various heat transfer equipments. Several investigations have been carried out on natural convection heat transfer and fluid flow with corrugated surfaces. (Chinnappa,1970) carried out an experimental investigation on natural convection heat transfer from a horizontal lower hot V-corrugated plate to an upper cold flat plate. He took data for a range of Grashof numbers from 10^4 to 10^6. Randall et al., 2000 studied local and average heat transfer coefficients fro natural convection
between a V-corrugated plate and a parallel flat plate to find the temperature gradient to estimate the local heat transfer coefficient. Local valued of heat transfer coefficient were investigated over the entire Vee-corrugated surface area. Using control volume based finite element method; Ali and Husain, 1992 investigated the natural convection heat transfer and flow characteristics in a square duct of V-corrugated vertical walls. Ali and Husain, 1993 also investigated the effect of corrugation frequencies on natural convective heat transfer and flow characteristics in a square enclosure of vee-corrugated vertical walls. This investigation showed that the overall heat transfer through the enclosure increases with the increase of corrugation for low Grashof number; but the trend is reversed for high Grashof number. Later Ali and Ali, 1994 carried out a finite element analysis of laminar convection heat transfer and flow of the fluid bounded by vee corrugated vertical plated of different corrugation frequencies. An enclosure with corrugated bottom surface maintaining a uniform heat flux and flat isothermal cooled top surface and side walls adiabatic wad studied by Noorshahi et al., 1992. The results showed that the pseudo-conduction region is increased with increase of wave amplitude. Yao, 1983 has studied theoretically the natural convection along a vertical wavy surface. He found that the local heat transfer rate is smaller than that of the flat plate case and decreased with increase of the wave amplitude. The average Nusselt number also shows the same trend. Adjlout et al., 1997 reported a numerical study of the effect of a hot wavy wall in an inclined differentially heated square cavity. Tests were performed for different inclination angles, amplitudes and Rayleigh numbers for one and three undulation. The trend of the local heat transfer is wavy.

In this investigation, a natural convection problem has been solved for different corrugation geometry and air has been taken as the working fluid. The corrugation geometry and the coordinate systems are shown in Figure 1 below. It consists of a vee-corrugated enclosure and a sinusoidal enclosure of dimension, W×H. In this work, two vertical walls are maintained at a constant temperature Tc, a constant heat flux, q is discretely embedded at the bottom wall, and the remaining parts of the bottom surface and the upper wall are considered to be adiabatic. The aspect ratio of the enclosure is defined as A = H / W and the corrugation amplitude has been fixed at 4% of the enclosure height. The Grashof number, Gr is varied from 10^3 to 10^6, the ratio of the heating element to the enclosure width, $\varepsilon = L/W$ and is taken as 0.4 and Prandtl number, Pr is taken as 0.71.

Mathematical Model

Natural convection is governed by the different equations expressing conservation of mass, momentum and energy. In the present study, we consider a steady two-dimensional laminar flow of incompressible fluid. The viscous dissipation term in the energy equation is neglected. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changed and to couple in this way the temperature field to the flow field. Then the governing equations for steady natural convection can be expressed in the dimensionless form as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Gr\,\theta \qquad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (4)$$

where U and V are the velocity components in the X and Y directions, respectively, θ is the temperature, P is the pressure, and Φ is the inclination angle of the enclosure with the horizontal direction, Gr and Pr are the Grashof number and Prandtl number, respectively, and they defined as:

$$Gr=\frac{g\beta\Delta T W^3}{\upsilon^2} \quad \text{and} \quad Pr=\frac{\upsilon}{\alpha} \qquad (5)$$

The dimensionless parameters in the equations above are defined as follow:

$X=\frac{x}{W}, Y=\frac{y}{W}, U=\frac{uW}{\upsilon}, V=\frac{vW}{\upsilon}, P=\frac{pW^2}{\rho\upsilon^2}, \theta=\frac{T-T_c}{\Delta T}$ and $\Delta T=\frac{qW}{k}$ where ρ, β, υ, α and g are the fluids density, coefficient of volumetric expansion, kinematic viscosity, thermal diffusivity, and gravitational acceleration, respectively. The corresponding boundary conditions for the above problem are given by:

Table 1: Comparison of the results for various grid dimensions at Gr = 10^6 (Vee corrugated)

Elements	1688	1976	2962	3526	4624	4788	6240
Nu	10.33007	10.33791	10.33934	10.34225	10.34243	10.34254	10.34255

Table 2: Comparison of the results for various grid dimensions at Gr = 10^6 (Sinusoidal corrugated)

Elements	1664	1972	2900	3770	4574	4740	6116
Nu	10.34106	10.33851	10.33622	10.33540	10.33160	10.33025	10.329838

Table 3: Average Nusselt number on the heated surface

Grashof number	**Nusselt number**	
	Vee corrugation	**Sinusoidal corrugation**
10^3	4.169	4.210
10^4	4.208	4.246
10^5	5.946	5.930
10^6	10.343	10.330

All walls: $U = V = 0$, Top wall: $\frac{\partial\theta}{\partial Y} = 0$, Right and left vertical walls: $\theta = 0$

Bottom wall:

$$\frac{\partial\theta}{\partial Y} = \begin{cases} 0 & \text{for} \quad 0 < X < 0.5(1-\varepsilon) \text{ and } 0.5(1+\varepsilon) < X < 1 \\ -1 & \text{for} \quad 0.5(1-\varepsilon) \le X \le 0.5(1+\varepsilon) \end{cases}$$

The average Nusselt number can be written as,

$$Nu = \frac{1}{\varepsilon}\int_0^{\varepsilon} \frac{1}{\theta_S(X)} dX \qquad (6)$$

where $\theta_S(X)$ is the local dimensionless temperature. The Simpson's 1/3 rule is used for numerical integration to obtain the average Nusselt number.

Numerical Procedure

The set of nonlinear ordinary differential equations (1)-(4) with boundary conditions are non-linear and coupled. It is difficult to solve them analytically. Hence we adopt an iterative scheme to obtain the solution numerically. The application of this technique is well documented Reddy, 1993. The continuity equation (1) will be used as a constraint due to mass conservation and this constraint may be used to obtain the pressure distribution (Reddy,1993) In order to solve Eqs. (2) - (4), we use the Penalty finite element method (Natarajan et al., 2008) where the pressure P is eliminated by a penalty parameter. The three noded triangular elements are used in this paper for the development of the finite element equations. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion δ such that

$$\left|\frac{\Gamma^{n+1} - \Gamma^{n}}{\Gamma^{n+1}}\right| < \delta \qquad (7)$$

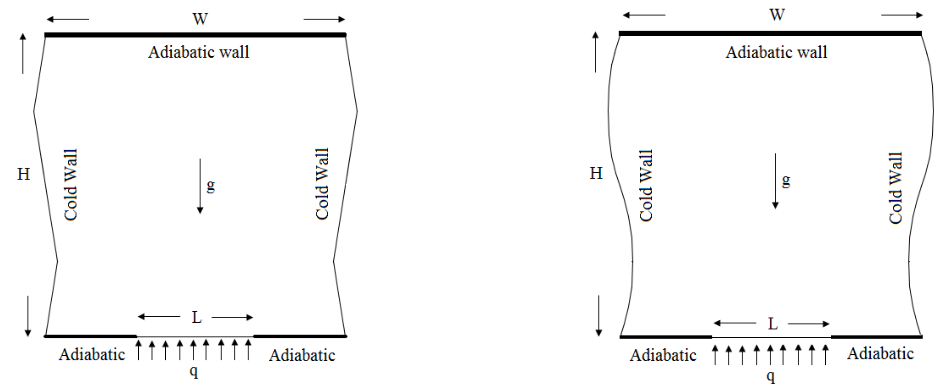
where n is the Newton iteration index and Γ = U, V, P and θ. The convergence criterion set to 10^{-6}.

To test and assess grid independence of the present solution scheme, many numerical runs are performed for higher Grashof number as shown in Table 1 and 2 above. These experiments reveal that a non-uniform spaced grid of 4788 elements for vee corrugated geometry and 4740 elements for sinusoidal geometry are adequate to describe correctly the flow and heat transfer process inside the enclosure. In order to validate the numerical model, the results are compared with those reported by Sharif and Mohammad (Natarajan et al., 2008), for square enclosure with Gr = 10^6 and ε = 0.4. In Figure. 2 below, a comparison of the isotherm plots for Gr = 10^6 and ε = 0.4 of the square enclosure is presented. The agreement is found to be excellent which validates the present computations indirectly.

RESULTS AND DISCUSSION

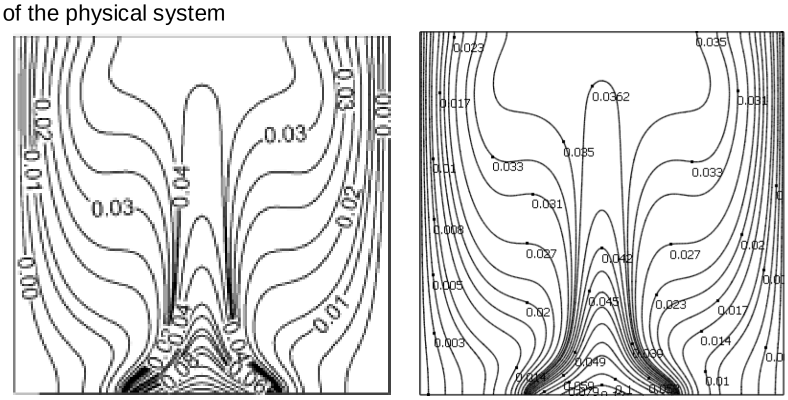
In this investigation, the average Nusselt number

Figures



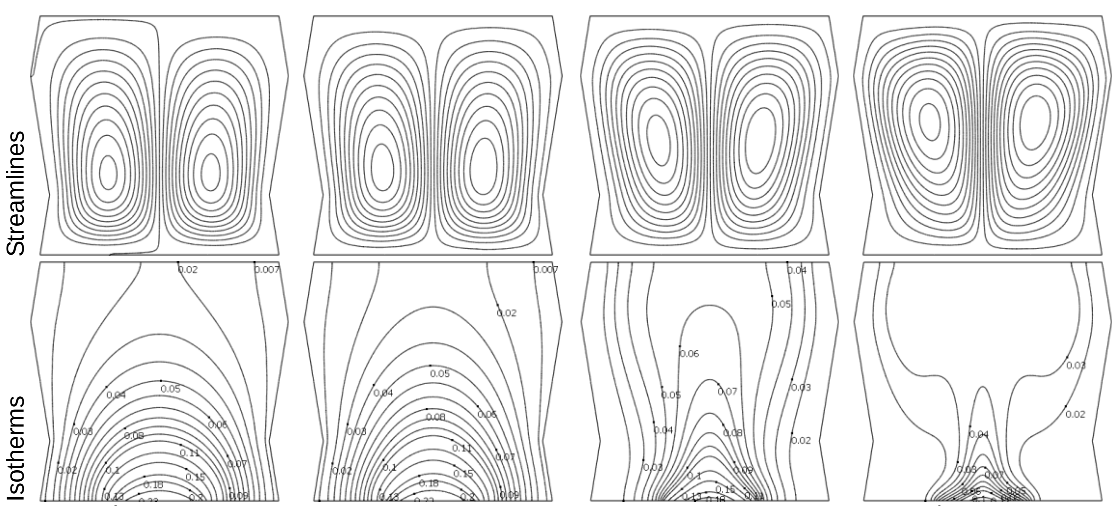
(a) Vee corrugated (b) Sinusoidal corrugated

Figure 1: Schematic diagram of the physical system



Sharif and Mohammad [9] Present Work

Figure 2: Comparison of the isotherm plots of the square straight enclosure with Sharif and Mohammad [9] at Gr = 10^6 and ε = 0.4



Gr = 10^3 Gr = 10^4 Gr = 10^5 Gr = 10^6

Figure 3: Streamlines and isotherms in the enclosure for vee corrugated vertical walls

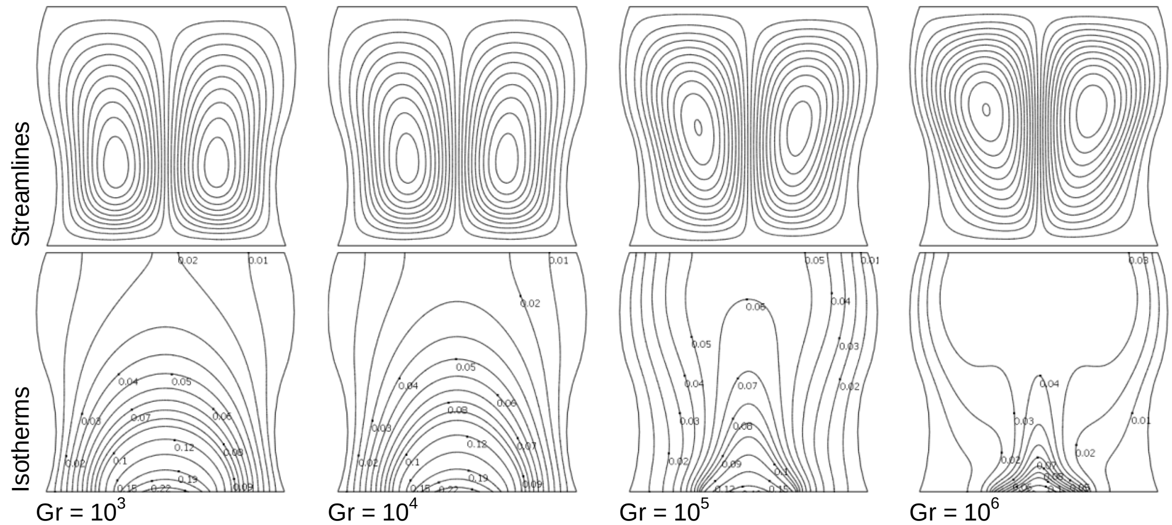


Figure 4: Streamlines and isotherms in the enclosure for sinusoidal vertical walls

along the hot wall is examined with respect to Grashof numbers 10^3, 10^4, 10^5 and 10^6 for different shapes of corrugated walls. The corrugation amplitude is fixed at 4% of the enclosure. The working fluid is chosen as air with Prandtl number, Pr = 0.71.

The main characteristics of the flow and energy transport are shown in the figures 3 and 4 in terms of streamlines and isotherms respectively for various Gr = 10^3 to 10^6. Visual examination of streamlines and isotherms does not reveal any significant difference among the different corrugation geometry. Because of the symmetrical boundary conditions on the vertical walls, the flow and temperature fields are symmetrical about the midline of the enclosure. The symmetrical boundary conditions in the vertical direction result in a pair of cells in the left and right halves of the enclosure. Because of symmetry, the flows in the left and right halves of the enclosure are identical except for the sense of rotation. As expected due to the cold vertical walls, fluids rise up from the middle portion of the bottom wall and flow down along the two vertical walls forming two symmetric rolls with clockwise and anticlockwise rotations inside the enclosure. The isotherms plots are also symmetrical about the vertical mid plane, the temperature decreases from the bottom to the top along the center line of the enclosure and concentrated towards the hot surface indicating the presence of large temperature gradient there. At Gr = 10^3, the heat transfer is purely due to conduction. During conduction dominant heat transfer, the temperature contours with θ = 0.01 – 0.03 occur symmetrically near the side walls of the enclosure. The other temperature contours with $\theta \geq 0.04$ are smooth curves symmetric with respect to the vertical symmetric line. At Gr > 10^4, the temperature gradients near both the bottom and side walls tend to be significant to develop the thermal boundary layer. Due to greater circulation near the central core at the top half of the enclosure, there are small gradients in temperature at the central regime whereas a large stratification zone of temperature is observed at the vertical symmetry line due to stagnation of flow. The thermal boundary layer develops partially within the cavity for Gr = 10^3 whereas for Gr = 10^5, the isotherms indicate that the thermal boundary layer develops almost throughout the entire enclosure. The isotherms patterns changes significantly with increasing Gr, indicating that the convection is the dominating heat transfer mechanism in the enclosure.

CONCLUSION

The effect of corrugation geometry on natural convection was investigated and analyzed numerically for different Grashof number and the streamline and isotherm plots were presented. For low Grashof number, the sinusoidal corrugation showed higher increment of Nusselt number than that of vee corrugation, but the trend is reversed for high Grashof number. Also the average Nusselt number increases with the increase of Grashof number. It can be pointed out that the decreased nature of heat transfer rate for corrugated geometry may be applied in practical situation where heat transfer reduction is desired across large temperature differences.

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