International Journal of Agricultural Research and Review: ISSN-2360-7971. 2(6): pp 070-075, June, 2014.

Full Length Research Paper

Allocation optimal water in irrigation network by Mixed- Integer Linear Programming Model (Case study: Sistan region of Iran)

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Accepted 11th May, 2014

In this study, Integer Linear Programming has been applied for optimal water distribution among irrigation networks. The case study is Sistan region in Sistan and Balouchistan Province of Iran. This model minimizes the number of channels which distribute water among outlets as well as difference between the goal and target time. Results provide a model for water distribution among irrigation networks based on time duration for water need and requested time for taking water. This model can be used to decide about irrigation planning.

Keywords: Irrigation networks, integer linear programming model, Sistan, Iran.

INTRODUCTION

Irrigation systems that deliver water to many small farm outlets can be difficult to manage, especially when farmers Request water at varying times in each irrigation scheduling period. One method to minimize management effort is to set a fixed flow in the supply channel at the head works at the start of the irrigation period and then to organize farmers to take water from the canal at agreed times. (Shahraky 2002) in his study based on the latest cultivation level under transmission network and nominal capacity examined the good division management, for water distribution and changing cultivation pattern in Sistan region. At first, he analyzed using scientific methods for water distribution network. Then he studied current water distribution among irrigation networks and by using scientific criteria did necessary assessments. (Monem and Namdarayan, 2002) used numerical method in SA optimization as multi-objective and presented a model for optimal distribution of water in irrigation canals and tested it on a canal in Varamin irrigation network (Needham et al., 2000) the integer linear programming method used in the operation of flood control in the river Iowa. And also (Srinvasan et al., 1999) used to optimization reservoir. (Ghaderi et al., 2009) developed software on base mixed integer linear ehran ming method for operation of reservoir multi objective plain ehran – karaj including dams Lar, Latian and Karaj. (Anwar and Clarke 2001) in your study used of A mixed-integer program that is presented for scheduling canal irrigation among a group of users where the duration of flow of each outlet and a target start time is specified by the users. In this study, the Integer linear Programming model applied for distribution of water among the various networks in the region of Sistan.

METHODOLOGY

This study is about delivering water among multiple outlets. In this way, consider waste water while at the same times for each outlet, after demanding, water is provided and minimized water loss in the minimum time.

Decision-variables

Any stream tube can supply any outlet. To define which stream tube supplies which outlet, a stream tube decision variable is introduced. This dimensionless decision variable Xij, is a binary integer with i being an index representing a stream tube and j being an index representing an outlet. Then

Xij =1 if stream tube i supplies outlet j=0 otherwise (1)

The second variable is Pijk that showed for a group of outlets supplied by any one stream tube; these outlets can be supplied in any sequence to define the sequence with which these outlets should receive water. Where i and j integers are as defined for (1) and k is an index representing on outlet. Then

Pijk=1 if stream tube i supplies outlets j and k and j is preceded by k =0 otherwise

(2)

The third decision variable is the scheduled start – time decision variable Sj. where the subscript j is an index representing the outlet. This decision variable is a nonnegative continuous variable measured in time unit, typically days or hours. For the purpose of these models, the origin of time (i.e., zero time) is

The start of the irrigation period and all points in time are measured relative to this origin. The scheduled start-time variable is introduced because, although the precedence decision variable will indicate the sequence of a group of outlets, an outlet may not necessarily start immediately after the preceding

Outlet has finished there may be slack time between outlets. To accommodate slack time between outlets, it is necessary to introduce the scheduled start-time decision variable. Objective function

The objective function is a dual-goal objective function. The first goal is to find the minimum number of stream tubes that can be used to provide water to every outlet for the duration specified by each outlet within the given irrigation period. The second goal is to find the sequence and scheduled start time

of each outlet so as to minimize the sum over all outlets of the difference between target start time and scheduled start time—within the number of stream tubes of the first goal. The objective function can be written as:

$$\min imizeZ = \theta.\sum_{i=1}^{n}\Phi_i + \sum_{j=1}^{n} L_j \qquad (3)$$

Where Z = objective function variable for single-period dual goal program; θ= priority factor; ϕ_i= activation function for stream tube i; and L_j = lead/lag time, i.e., the absolute difference between scheduled start time and target start time for outlet j. The priority factor is assigned to the first objective to give priority to the goal of minimizing the number of stream tubes over minimizing the total lead/lag time. The priority factor needs to be chosen carefully, taking into account computational efficiency (*LINGO* 1999) and also the accuracy of numerical computers.

Constraints

Every outlet must receive water from one and only one stream tube. This constraint is enforced by

$$\sum^{n} X_{ij} = 1 \quad \text{for every} \qquad (4)$$

Where N = number of outlets. Reiterating, one extreme solution of the model would be for the number of stream tubes to be equal to the number of outlets. hence the use of the number of outlets in (4). It is also acceptable to arbitrarily select an integer smaller than the number of outlets. This has the effect of reducing computation time. However, if the integer selected is too small, the model will become infeasible; i.e., for the given maximum number of stream tubes, it is not possible to supply all the outlets with the durations specified for each outlet. This is a subtle although important point and can be used to reduce the computation time of the model.

Defining σ_i as the total number of outlets receiving water from the *i*th stream tube or

$$\sigma_i = \sum_{j=1}^{n} X_{ij} \text{ for every } I \quad (5)$$

Then, for any stream, the activation function Φ_i takes a value of zero if σ_i is zero and one otherwise, hence

$$\Phi_i = 0 \quad \text{if} \quad \sigma_i = 0 \quad =1 \quad \text{if} \quad \sigma_i \geq 1 \quad (6)$$

The conditions in (6) can be expressed by defining Φ_i as a binary integer and introducing the following single linear constraint (Taha 1976):

$$C_i \times \Phi_i \geq \sigma_i \quad (7)$$

Where C_1 = any suitable positive constant greater than the largest possible value of σ_i. Schrage (1999) recommends that, for reasons of computational efficiency, *C*1 should be chosen with care and should not be excessively large. Using (7) rather than (6) avoids the difficulties caused by the discontinuous variable in (6).

The first constraint on the precedence decision variable is that if stream tube *i* feeds outlets *j* and *k*, and *j* precedes *k*, then the reverse cannot be true, i.e., *k* cannot precede *j*. This condition is enforced by the constraint

$$P_{i,j,k} + P_{i,k,j} \leq 1 \text{ For every i, j, k} \quad (8)$$

For an outlet *j* that is fed by channel *i*, outlet *j* can be immediately preceded by at most one instance of outlet *k*, provided *k* is also fed by stream tube *i*. If outlet *j* is the first outlet to be fed by stream tube *i*, then it will have no outlets immediately preceding it, hence the term "at most." This is enforced by:

$$\sum_{k=1}^{n} P_{i,j,k} \leq X_{i,j} \quad \text{For every j} \quad (9)$$

Similarly, any outlet *j* can have at most one outlet *k* immediately preceding it—again with the caveat that both *j* and *k* are fed by stream tube *i*, and

$$\sum_{j=1}^{n} P_{i,j,k} \leq X_{i,j,k} \quad \text{For every } k \quad (10)$$

The fourth constraint on the precedence decision variable is that for a given channel *i*, the sum of all outlets preceded by other outlets must be one less than the number of outlets fed by the stream tube *i*. This condition is enforced by the constra

$$\sum_{j=1}^{n}\left(\sum_{k=1}^{n} P_{I,J,K}\right) \leq \sigma_i - \Phi_i \text{ for every} \quad \text{i} \quad (11)$$

Which enforces the constraint that if there are a group of four outlets receiving water from the same stream tube, three of these outlets will be preceded by another outlet, i.e., number of outlets -1, or $(\sigma_i - 1)$. The right side of (11) is $(\sigma_i - \Phi_i)$ rather than $(\sigma_i - 1)$ because some channels will not feed any outlets, i.e. $(\sigma_i = 0)$, in which case the expression $(\sigma_i - 1)$ would evaluate to -1 and the solution would be infeasible. For $(\sigma_i = 0)$, from $(\Phi_i = 0)$ and therefore the right side of (11) evaluates to 0.

The first constraint on the scheduled start-time decision variable is that every outlet must complete its duration within the irrigation period, i.e., for every outlet the sum of scheduled start time and duration must be less than (or equal to) the irrigation period. This constraint is enforced by:

$$(S_j + D_j) \leq T_L \quad \text{for every } j \quad (12)$$

Where S_j = scheduled start time for outlet *j*; D_j = duration of the *j*th outlet; and T_L = irrigation period. The irrigation period is the time the canal is in service and can supply water to the outlets. This could be 85% of the irrigation interval, with 15% of the irrigation interval allowed for maintenance and inspection operations. Alternatively, maintenance and inspection operations could be restricted to an annual cycle, in which case the entire irrigation interval could be used for the irrigation period. Essentially the irrigation period is the period over which the outlets are to be supplied water.

For outlets receiving water from the same stream tube, an outlet can start at any time after the preceding one has finished, i.e., the scheduled start time of any outlet can be greater than or equal to the sum of the start time of the preceding outlet and the duration of the preceding outlet. In terms of the variables of the model, this constraint is given by

$$S_j \geq \sum_{i=1} \left\{ \sum_{k=1}^{N} \left[P_{i,j,k} \times (S_k + D_k) \right] \right\}$$ For every *j* given that j≠ *k* (13)

Or

$$S_j \geq \sum_{i=1}^{n} \left\{ \sum_{k=1}^{N} \left[(P_{i,j,k} \times S_k) + (P_{i,j,k} \times D_k) \right] \right\}$$ For every *j* given that *j* ≠*k* (14)

In (14) the product $(P_{i,j,k} \times S_k)$ is a product of a binary integer and a nonnegative continuous variable. This product of variables can be transformed into linear expression by introducing the inequalities

$$\gamma_{i,j,k} \leq S_k$$
$$\gamma_{i,j,k} \leq M_y \times P_{i,j,k} \qquad (15)$$
$$\gamma_{i,j,k} \geq S_k - M_x \times (1 - P_{i,j,k})$$

Where $\gamma_{i,j,k} = P_{i,j,k} \times S_k$; and M_x and M_y = upper bounds on the values of S_k and $(P_{i,j,k} \times S_k)$, respectively. This transformation technique of a product of a binary integer and a continuous variable is explained in more detail in Schrage (1999).

Having defined all the constraints for all three decision variables, the lead/lag time can now be defined. The lead/lag time is the absolute difference between the scheduled start time for each outlet and the target start time for that outlet. This is defined by

$L_j = S_j - G_j$ If $S_j > G_j$

or

$L_j = G_j - S_j$ If $S_j \leq G_j$

(16)

Where G_j = target start time as requested by user for outlet *j*. The variable L_j can be represented by the absolute function as

$$L_j = |S_j - G_j| \quad \text{For every } j \qquad (17)$$

But as Schrage (1999) pointed out, this is computationally unwise since the absolute function is discontinuous. The use of the absolute function can be avoided by introducing variables α_j and β_j and the inequalities

$$\beta_j \geq G_j - S_j \quad \text{For every } j \qquad (18)$$

Where α_j and β_j = positive variables representing absolute difference between scheduled and target start times. Constraint (18) can be represented as

$$\alpha_j - \beta_j = S_j - G_j \quad \text{For every } j \qquad (19)$$

And (17) can be represented by

$$L_j = \alpha_j + \beta_j \quad \text{For every } j \qquad (20)$$

Thereby removing the need for using the absolute function

FINDINGS

To demonstrate practical application of this model, the model is applied to sistan irrigation project. Sistan region of Iran has three main irrigation areas which are Zahak, Miankangi and Chahnimeh irrigation networks. Miankangi irrigation network is watered through three watercourses of Shirdel, Canal 1 of Miankangi and Golmir River off Common Parian. Zahak irrigation network is fed by Hirmand River through Sistan River. This network is divided into three branches of Taheri creek, Shahr creek and Sistan River at the location of Zahak Dam. In Chahnimeh irrigation network, water is supplied through Chahnimeh reservoirs. This network supplies water requirements of Shib-Ab and Posht-Ab regions. Shib-Ab and Posht-Ab networks each divide into five number two sub-canals.

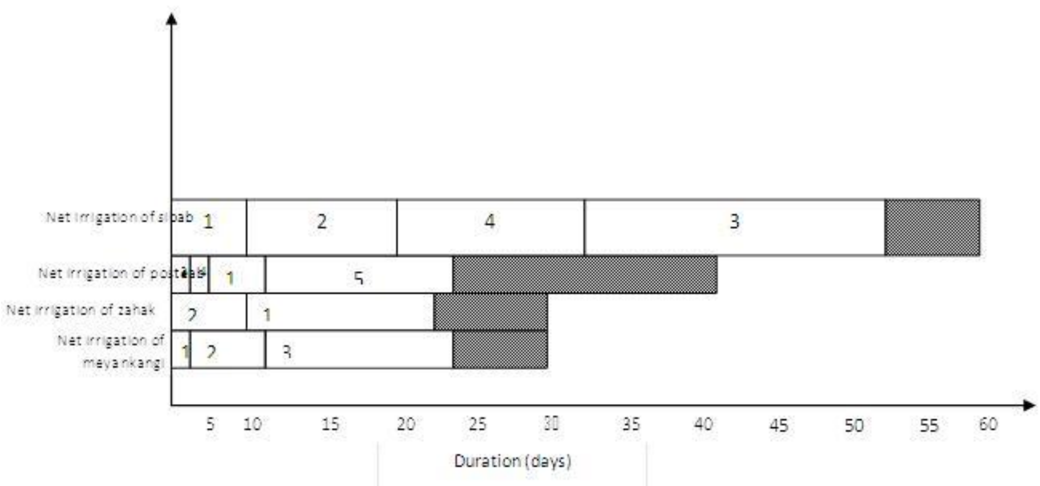
Each outlet with a requested duration of flow as given in Table 1. The outlets do not have any target start times, and therefore, for the purpose of this application they are generated

Randomly, using a spreadsheet. . A priority factor of 100 was selected. This priority factor makes the minimum number of stream tubes several orders of magnitude greater than the sum of lead/lag over all outlets; hence priority is given to the goal of minimizing the number of stream tubes used. The value of 100 also allows easier interpretation of the objective function, as will be demonstrated subsequently. Irrigation period is considered 30 days for Miankangi and Zahak networks, 40 days for Posht-Ab network and 60

Table 1. Canal and outlets data with random target start time added

Channel	outlet	Number of outlet	Duration of flow (day)	Target start time (day number)**
Miankangy Network	Shirdel stream	1	4.9	0
	Golmir stream	2	5.3	0.25
	Miankangy No.1 channel	3	12.2	0.5
Zahak Network	Shahr channel	1	14.32	0
	Taheri channel	2	7.6	0.25
Poshtab Network	Channel Number.1	1	5.09	0
	Channel Number.2	2	12.183	0.25
	Channel Number.3	3	1.9	0.5
	Channel Number.4	4	4.3	0.75
	Channel Number.5	5	12.61	1
Shibab Network	Channel Number.1	1	9.3	0
	Channel Number.2	2	10.1	0.25
	Channel Number.3	3	19.9	0.5
	Channel Number.4	4	13.47	0.75

Note: **Target start time is considered 6 hours after origin time that is zero.



days for Shib-Ab network. In this study, canal capacities are assumed identical.

Eqs. (1)– (20) were formulated into a suitable input file and LINGO 8.0 was used as the general purpose solver. The optimum solution for canal irrigation meyankangi is found at an objective function in (3) of 114.100. The first digit of 1 indicates the minimized number of stream tubes necessary. The priority factor of 100 has multiplied the number of stream tubes to 100; therefore the model gives the stream-tube goal priority. The 14.1 in the objective function represents 14.1 days, the minimized sum of lead/lag time for all outlets. Also optimal solution for canal irrigation zahak is found of 107.85 that minimized number was 1 and 7.85 minimized sum of lead/lag time. For canal irrigation shib ab and posh tab respectively objective function value is 141.79 and 159.57, also the minimum number of channels for these two networks is only one channel. Schedule model shown in Figure 1 and table 2 is a discharge-duration diagram of the optimum solution from the model. Each block in Figure 1 represents the actual start time and duration of each outlet. The target start time for each outlet has also been represented on Figure 1 and table 2.

In Miankangi irrigation network sequence of

Table 2. results from solving the model

Channel	outlet	Number of outlet	Order water supply	Planning time (day)	Target start time (day number)
Miankangy network	Shirdel dtream	1	1	0	0
	Golmir stream	2	2	4.9	0.25
	Miankangy first chanel	3	3	10.2	0.5
Zahak Network	Taheri stream	1	2	0	0.25
	Shahr stream	2	1	7.6	0
Poshtab Network	Channel Number.3	1	3	0	0.5
	Channel Number.4	2	4	1.9	0.75
	Channel Number.1	3	1	6.2	0
	Channel Number.5	4	5	11.29	1
	Channel Number.2	5	2	23.9	0.25
Shibab Network	Channel Number.1	1	1	0	0
	Channel Number.2	2	2	9.3	0.25
	Channel Number.4	3	4	19.4	0.75
	Channel Number.3	4	3	32.87	0.5

Source: research finding

water received by each output is in accordance with the actual conditions. Canal 1 receives water at the same time that it demands, Canal 2 receives water 4.65 days after its demand and also Canal 3 receives water 9.7 days after its request. For other networks sequence of water received by outputs differs from actual conditions. In Zahak irrigation network, Canal 2 (Taheri creek) first receives water at zero time and after its request was completely satisfied Canal 1 (Shahr creek) receives water 7.6 days after its request. Shaded boxes in Figure 1 indicate the delay times for each network, which this delay time is 7.6 days for Miankangi network, 8.08 days for Zahak network, 3.27 days in Posht-Ab network and 7.23 days in Shib-Ab network.

SUGGESTION

A model can be provided for distribution of water between irrigation networks based on the period of time that they need water and the time of their demand for water, by using the results of this study. These results can also be used to decide on irrigation projects.

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